Time Frequency Analysis of Railway Polygonized Wheel Detection

2022.12.6

Outline

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  - Polygonized Wheel
- Method
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  2. Ensemble EMD + Wigner-Ville Distribution
- Results
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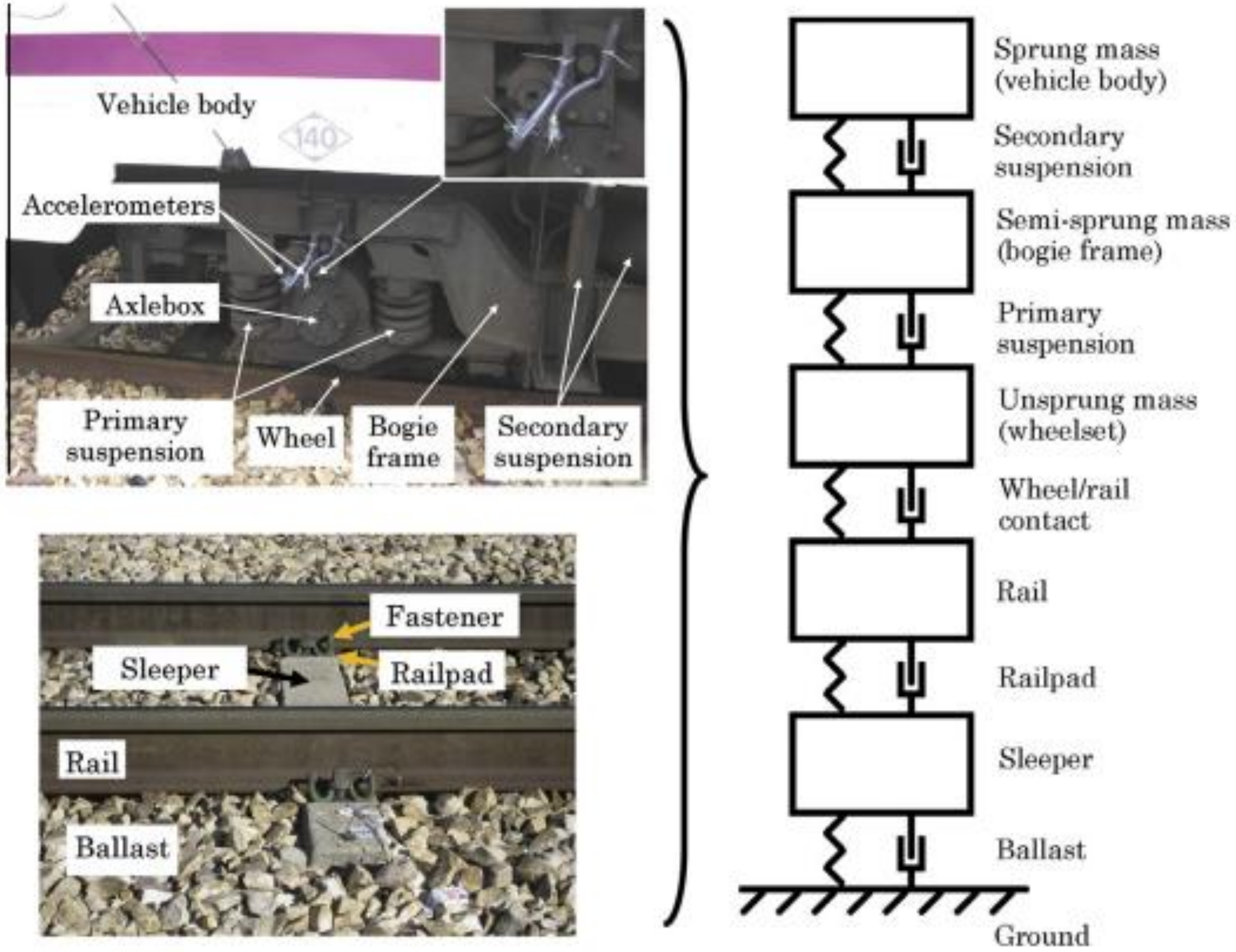
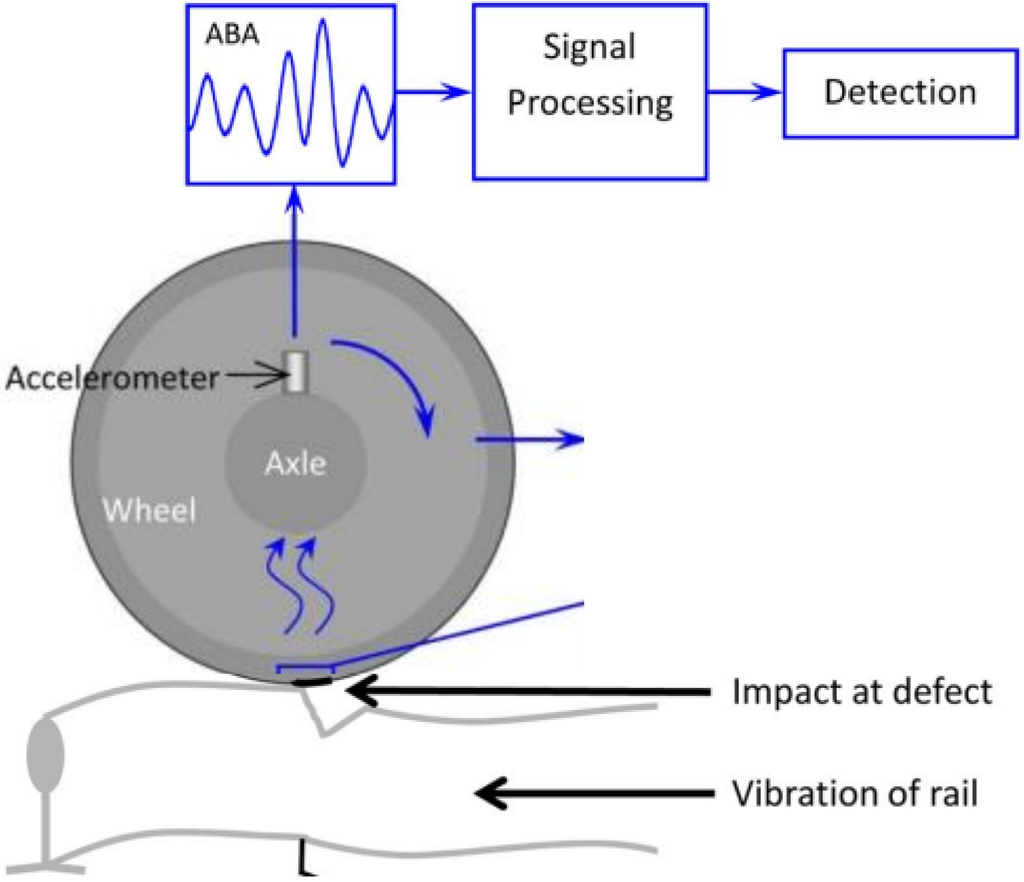
How to Measure?

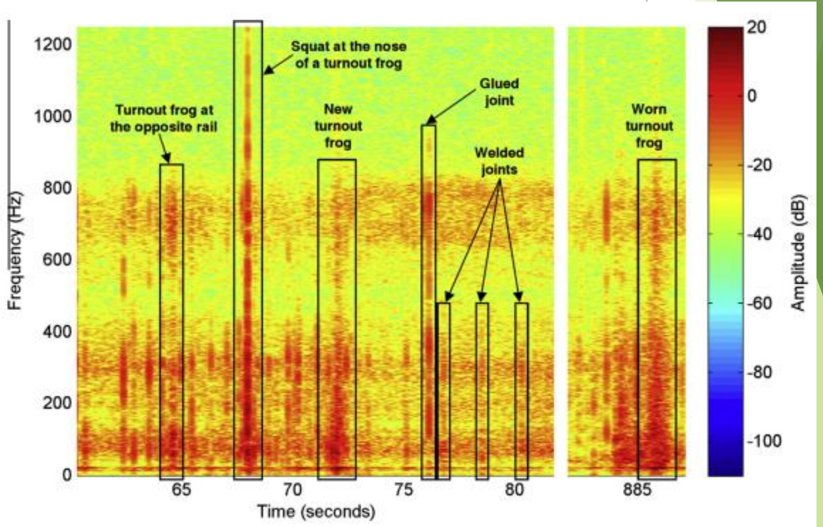


From Reference [2]

How to Measure?

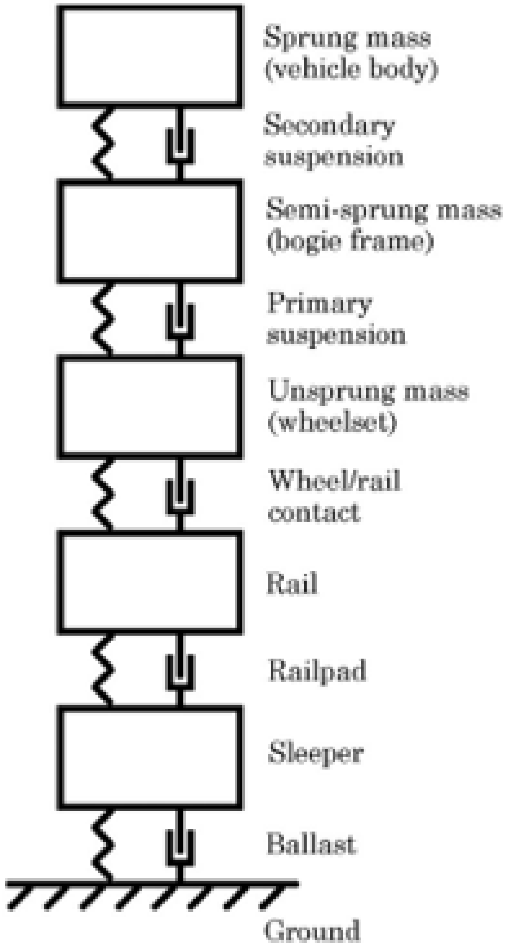


From Reference [3]

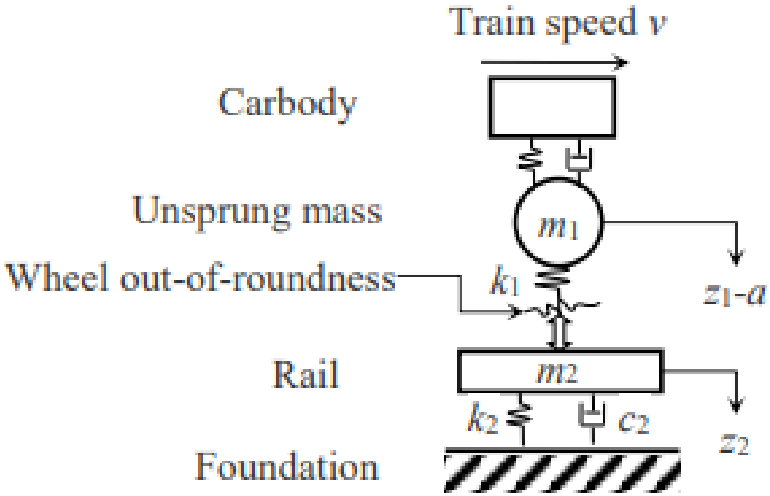


From Reference [2]

Polygonized Wheel

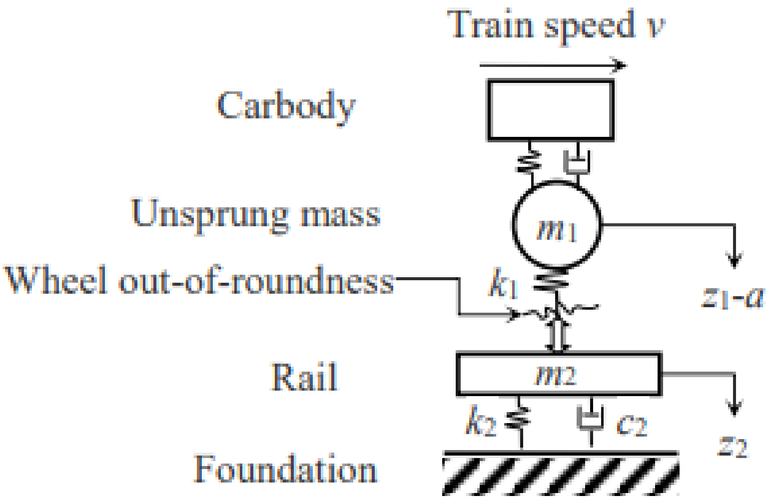


From Reference [2]



From Reference [1]

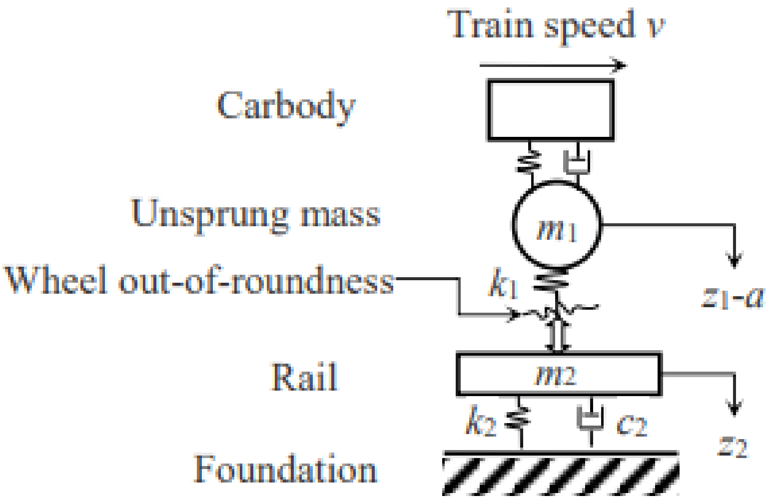
Polygonized Wheel



z_1: vertical displacement of wheel set k_1
z_2: vertical displacement of the track
a: out-of-roundness amplitude

$$\begin{cases} m_1\ddot{z}_1 + k_1(z_1 - a - z_2) = 0 \\ m_2\ddot{z}_2 + c_2\dot{z}_2 + k_2 z_2 - k_1(z_1 - a - z_2) = 0 \end{cases}$$

Polygonized Wheel



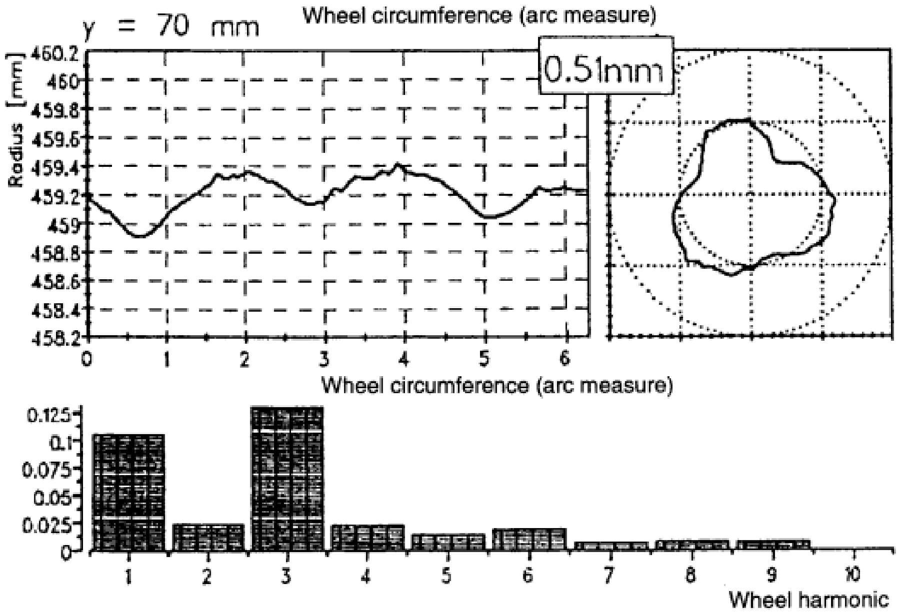
z_1: vertical displacement of wheel set k_1
z_2: vertical displacement of the track
a: out-of-roundness amplitude

$$\begin{cases} m_1\ddot{z}_1 + k_1(z_1 - a - z_2) = 0 \\ m_2\ddot{z}_2 + c_2\dot{z}_2 + k_2 z_2 - k_1(z_1 - a - z_2) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{\ddot{z}_1}{\omega_1^2} + z_1 - z_2 = a \\ \dfrac{\ddot{z}_2}{\omega_2^2} + \dfrac{c_2}{k_1}\dot{z}_2 + \left(\dfrac{k_1 + k_2}{k_1}\right) z_2 - z_1 = -a \end{cases}$$

where $\omega_1 = \sqrt{k_1/m_1}$ and $\omega_2 = \sqrt{k_2/m_2}$

Polygonized Wheel



From Reference [4]

z_1: vertical displacement of wheel set k_1
z_2: vertical displacement of the track
a: out-of-roundness amplitude

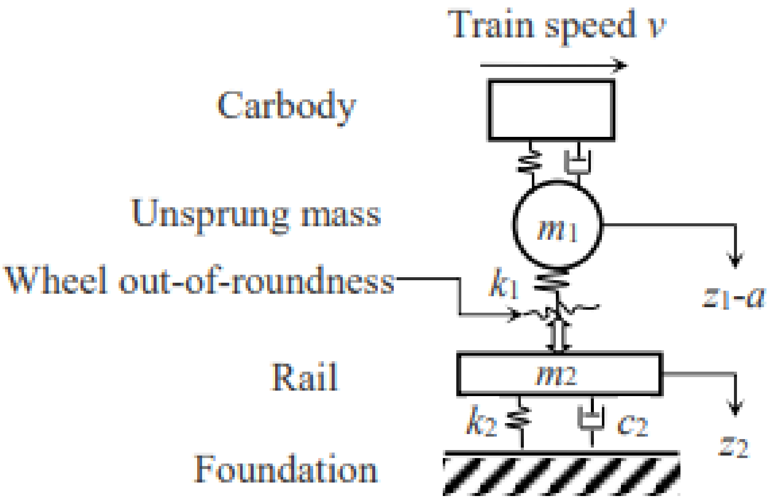
a$\rightarrow a(t) = a\sin(\omega t + \varphi)$

Laplace transform:

$$a(s) = \frac{a(s\sin\theta + \omega\sin\theta)}{s^2 + \omega^2}$$

$$\begin{cases} z_1(s) = H_{z1}(s)a(s) \\ z_2(s) = H_{z2}(s)a(s) \end{cases}$$

Polygonized Wheel



z_1: vertical displacement of wheel set k_1
z_2: vertical displacement of the track
a: out-of-roundness amplitude

Laplace transform of the solution:

$$\begin{cases} \left(\frac{s^2}{\omega_1^2}+1\right)z_1(s)-z_2(s)=a(s) \\ z_1(s)-\left(\frac{s^2}{\omega_2^2}+\frac{c_2 s}{k_1}+\frac{k_2}{k_1}+1\right)z_2(s)=a(s) \end{cases}$$

$$H_{z1}(s)=\frac{z_1(s)}{a(s)}=\frac{\frac{s^2}{\omega_2^2}+\frac{c_2 s}{k_1}+\frac{k_2}{k_1}}{\frac{s^4}{\omega_1^2\omega_2^2}+\frac{c_2 s^3}{k_1\omega_1^2}+\left[\frac{1}{\omega_2^2}+\left(\frac{k_2}{k_1}+1\right)\frac{1}{\omega_1^2}\right]s^2+\frac{c_2 s}{k_1}+\frac{k_2}{k_1}}$$

Polygonized Wheel

$$H_{z1}(s) = \frac{\frac{s^2}{\omega_2^2} + \frac{c_2 s}{k_1} + \frac{k_2}{k_1}}{\frac{s^4}{\omega_1^2 \omega_2^2} + \frac{c_2 s^3}{k_1 \omega_1^2} + \left[\frac{1}{\omega_2^2} + \left(\frac{k_2}{k_1} + 1\right)\frac{1}{\omega_1^2}\right] s^2 + \frac{c_2 s}{k_1} + \frac{k_2}{k_1}}$$

$$\ddot{z}_1(s) = s^2 z_1(s) = s^2 H_{z1}(s) a(s) = s^2 H_{z1}(s) \frac{a(s \sin\theta + \omega \sin\theta)}{s^2 + \omega^2}$$

$$\ddot{z}_1(s) = \frac{\frac{s^4}{\omega_2^2} + \frac{c_2 s^3}{k_1} + \frac{k_2}{k_1} s^2}{\frac{s^4}{\omega_1^2 \omega_2^2} + \frac{c_2 s^3}{k_1 \omega_1^2} + \left[\frac{1}{\omega_2^2} + \left(\frac{k_2}{k_1} + 1\right)\frac{1}{\omega_1^2}\right] s^2 + \frac{c_2 s}{k_1} + \frac{k_2}{k_1}} \frac{a(s \sin\theta + \omega \sin\theta)}{s^2 + \omega^2}$$

Routh-criterion

For a fourth-order polynomial $P(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0$, if

1. All the coefficients $a_i > 0$
2. $a_3 a_2 a_1 > a_4 a_1^2 + a_3^2 a_0$
3. $a_3 a_2 > a_4 a_1$

Then P(s) is stable.

Therefore, $\frac{s^4}{\omega_1^2 \omega_2^2} + \frac{c_2 s^3}{k_1 \omega_1^2} + \left[\frac{1}{\omega_2^2} + \left(\frac{k_2}{k_1} + 1\right)\frac{1}{\omega_1^2}\right] s^2 + \frac{c_2 s}{k_1} + \frac{k_2}{k_1}$ is stable.

Polygonized Wheel

$$H_{z1}(s) = \frac{\frac{s^2}{\omega_2^2} + \frac{c_2 s}{k_1} + \frac{k_2}{k_1}}{\frac{s^4}{\omega_1^2 \omega_2^2} + \frac{c_2 s^3}{k_1 \omega_1^2} + \left[\frac{1}{\omega_2^2} + \left(\frac{k_2}{k_1} + 1\right)\frac{1}{\omega_1^2}\right] s^2 + \frac{c_2 s}{k_1} + \frac{k_2}{k_1}}$$

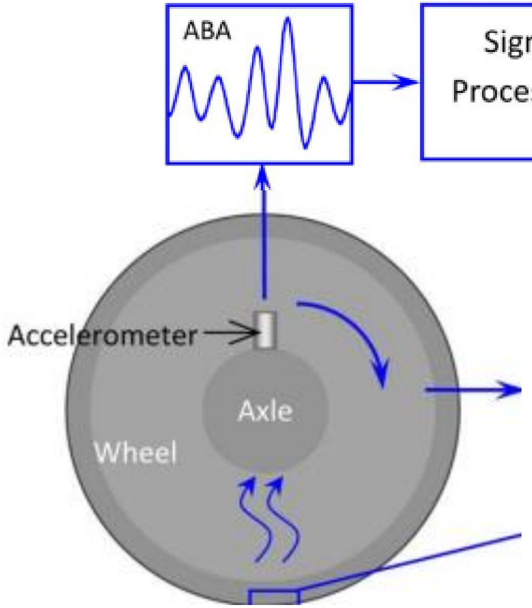
$$\ddot{z}_1(s) = s^2 z_1(s) = s^2 H_{z1}(s) a(s) = s^2 H_{z1}(s) \frac{a(s \sin\theta + \omega \sin\theta)}{s^2 + \omega^2}$$

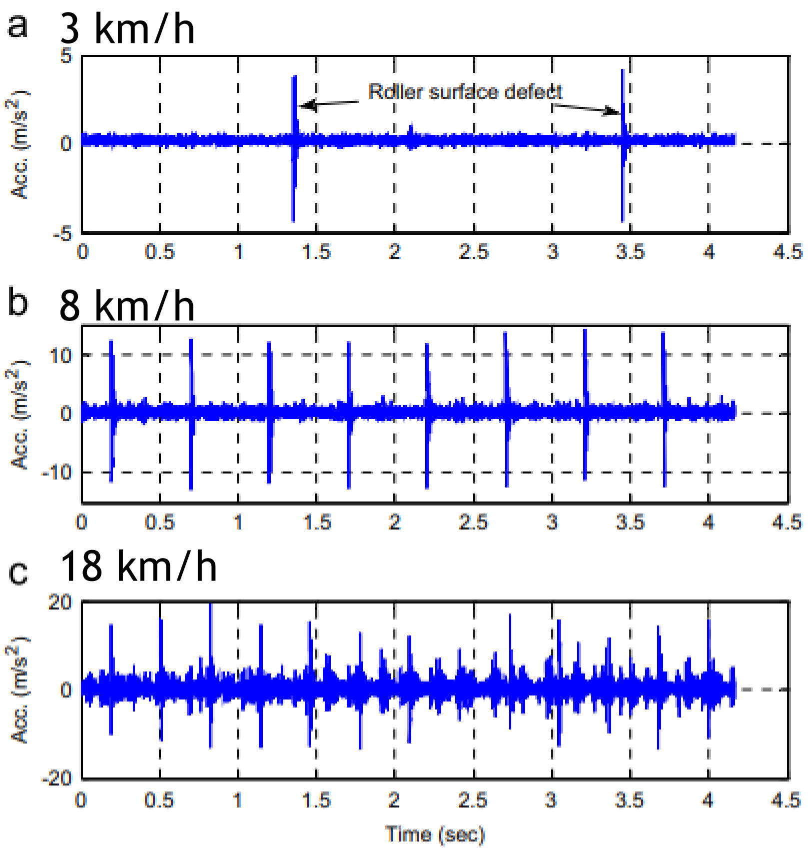
Since $\frac{s^4}{\omega_1^2 \omega_2^2} + \frac{c_2 s^3}{k_1 \omega_1^2} + \left[\frac{1}{\omega_2^2} + \left(\frac{k_2}{k_1} + 1\right)\frac{1}{\omega_1^2}\right] s^2 + \frac{c_2 s}{k_1} + \frac{k_2}{k_1}$ is stable by Routh-criterion, the necessary and sufficient for all roots having negative real parts is

$$\ddot{z}_1(s) = \sum_{i=1}^{4} \frac{A_i}{s + p_i} + \frac{Bs + C}{s^2 + \omega^2}$$

$$\ddot{z}_1 = \sum_{i=1}^{4} A_i e^{-p_i t} + D cos(\omega t + \varphi)$$

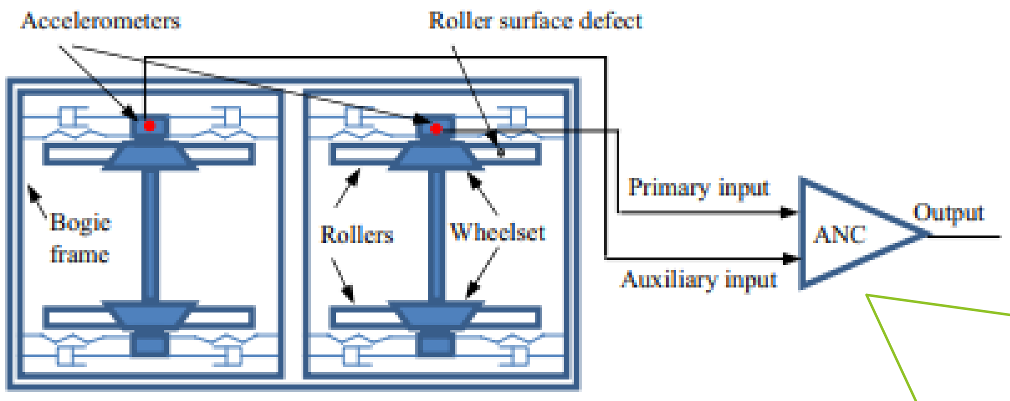
Adaptive Noise Cancellation

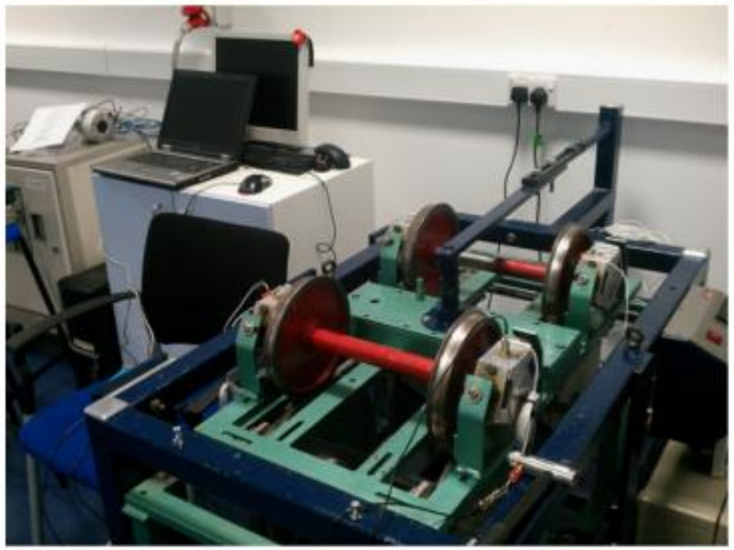


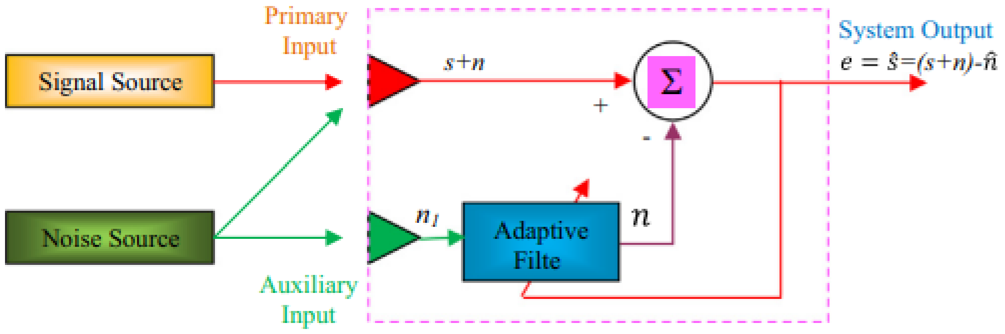


From Reference [5]

Adaptive Noise Cancellation

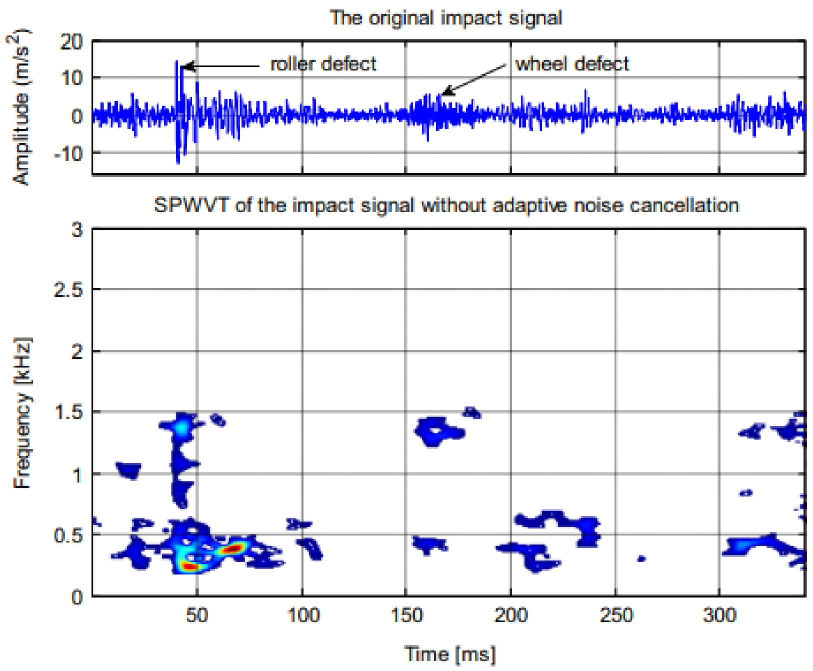




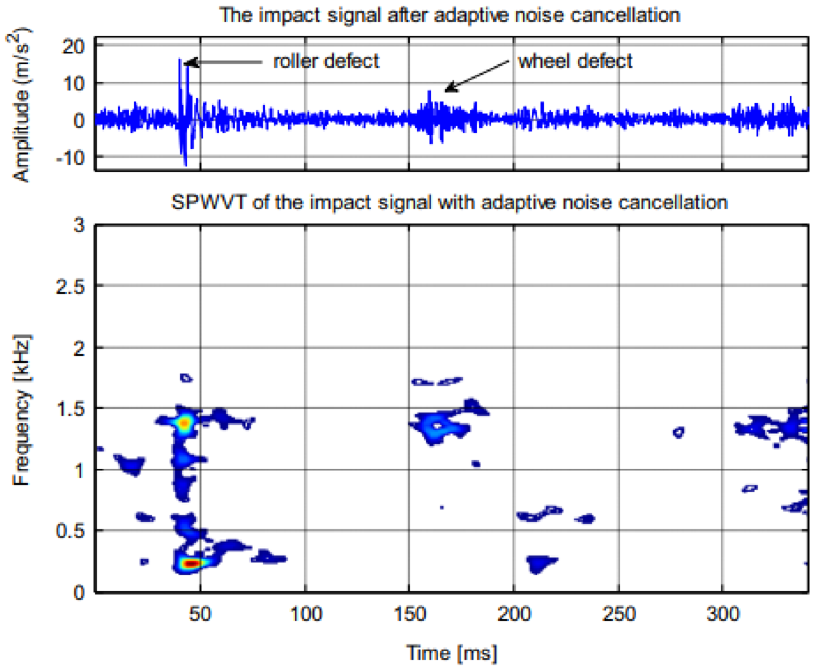


From Reference [5]

Adaptive Noise Cancellation



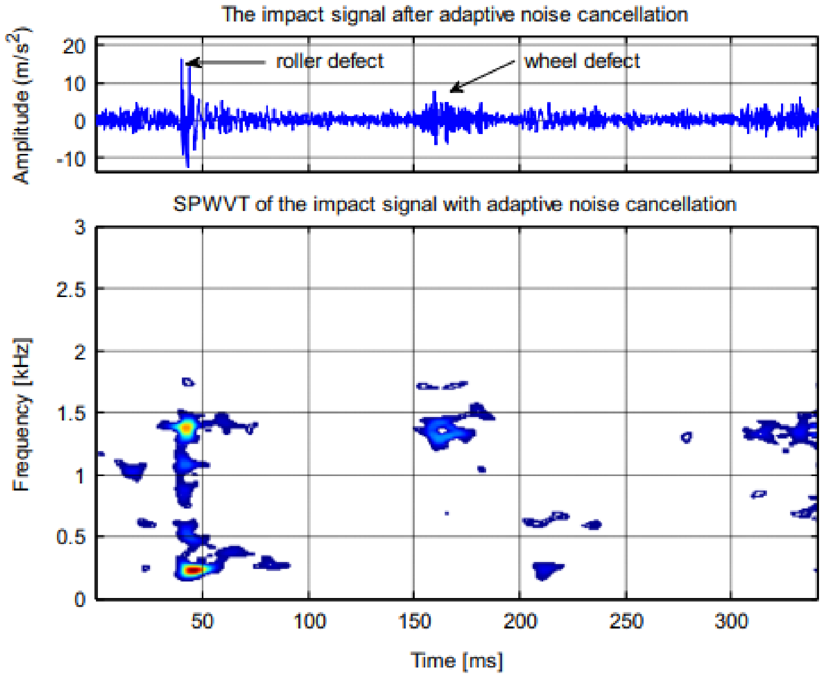
Without Noise Cancellation



With Noise Cancellation

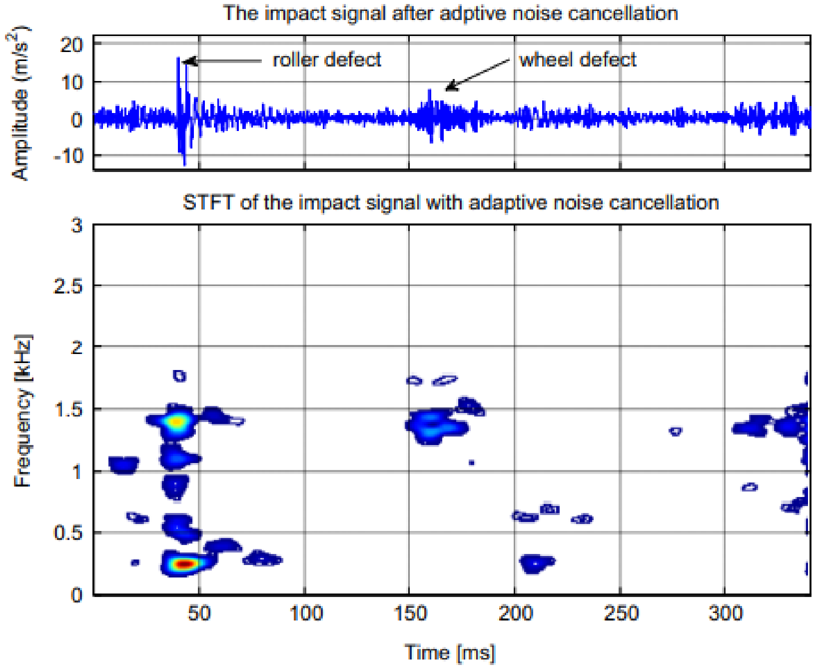
From Reference [5]

Wigner-Ville Distribution



Wigner-Ville Distribution

$$\int_{-\infty}^{\infty} s(t+\tau/2)s^*(t-\tau/2)e^{-j\omega\tau}d\tau$$

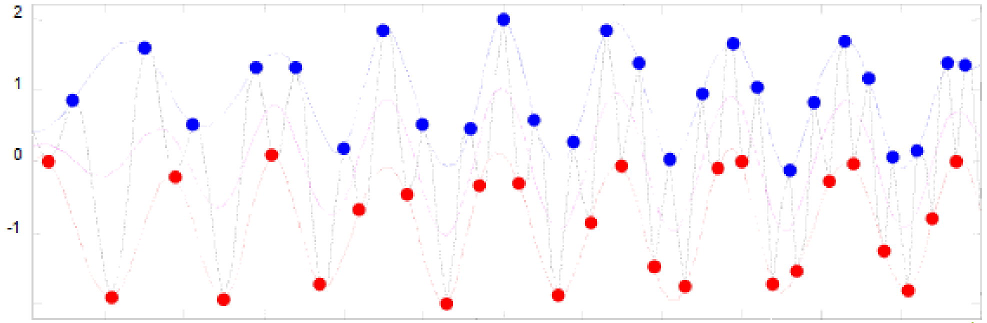


STFT

$$\int_{-\infty}^{\infty} h(\tau-t)s(t)e^{-j\omega t}dt$$

From Reference [5]

Ensemble EMD



$$h_k(t) = y(t) - \frac{e_{min}(t) + e_{max}(t)}{2}$$

Step 1:
Add noise

Step 2:
y(t)=x(t)

Step 3: do

Step 4:
$h_k(t)$ is an IMF?
1. Local maximum > 0
2. Local minimum < 0
3. $\left|\frac{u_1(t)+u_0(t)}{2}\right| < thres$

Y

Step 5:
$$x_0(t) = x(t) - \sum_{s=1}^{n} c_s(t)$$
has only 0 or 1 extreme?

Y

$y(t) = h_k(t)$ N

$y(t) = x_0(t)$ N

Add noise with different SNR

N

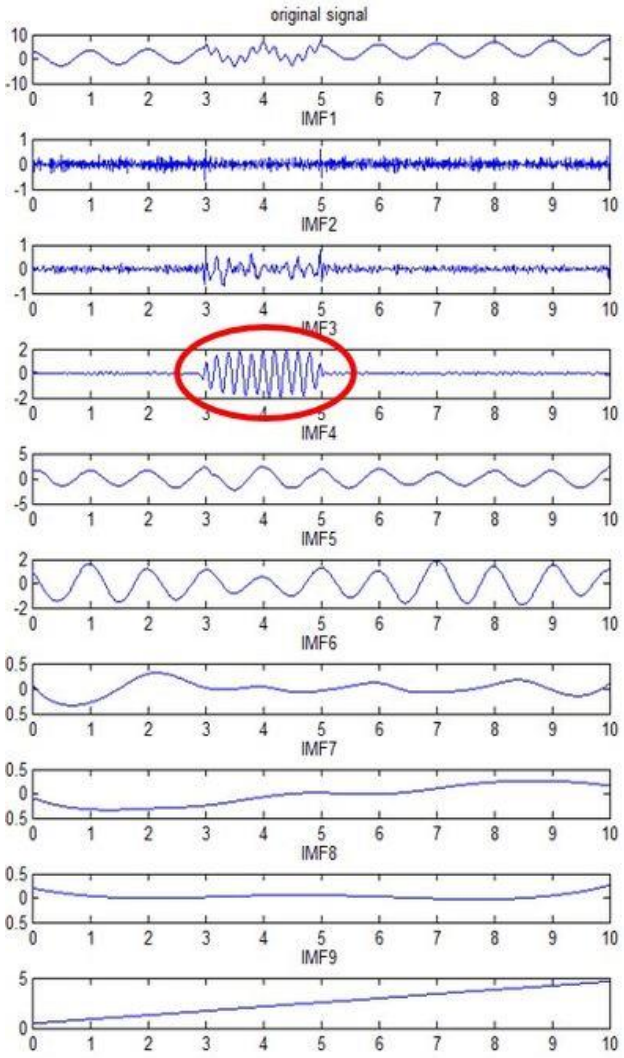
Step 6:
All IMFs are single frequency signals?

Y

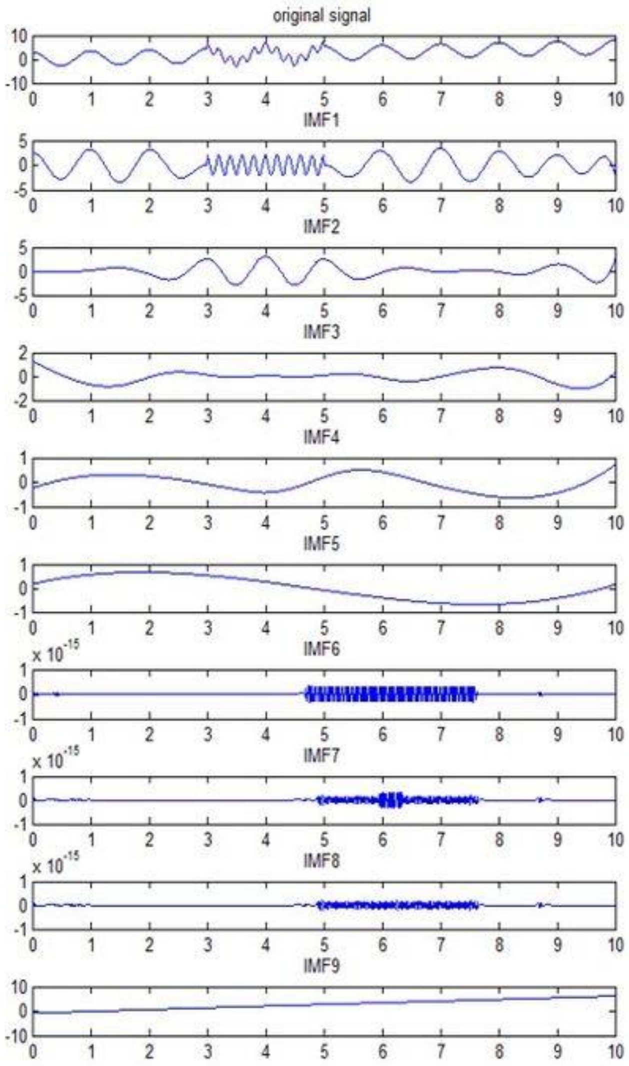
end

From Reference [6]

Ensemble EMD



Ensemble EMD

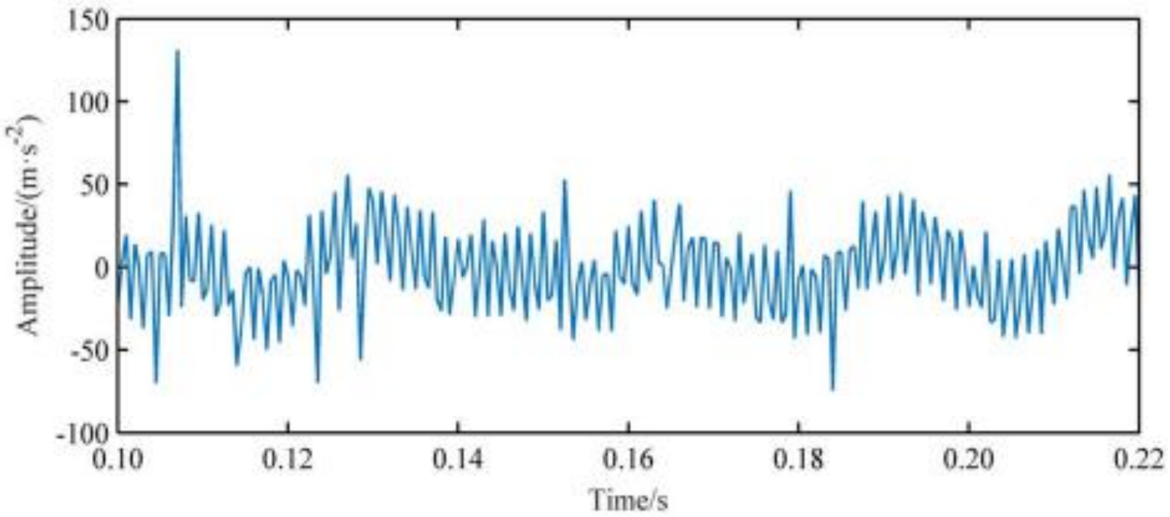


EMD

From Reference [7]

Results

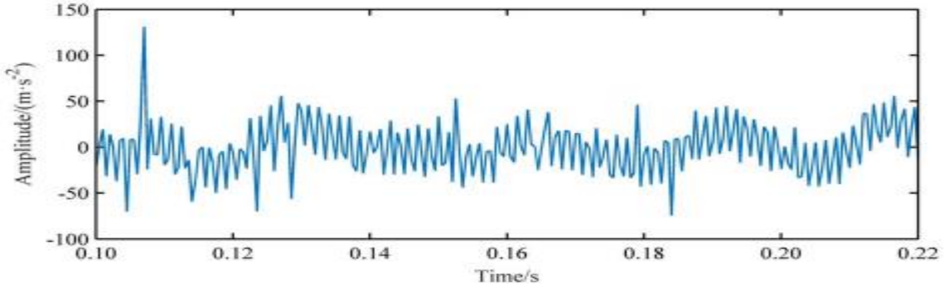
Raw vibration signal x(t) → Adaptive Noise Cancellation → EEMD → WVD



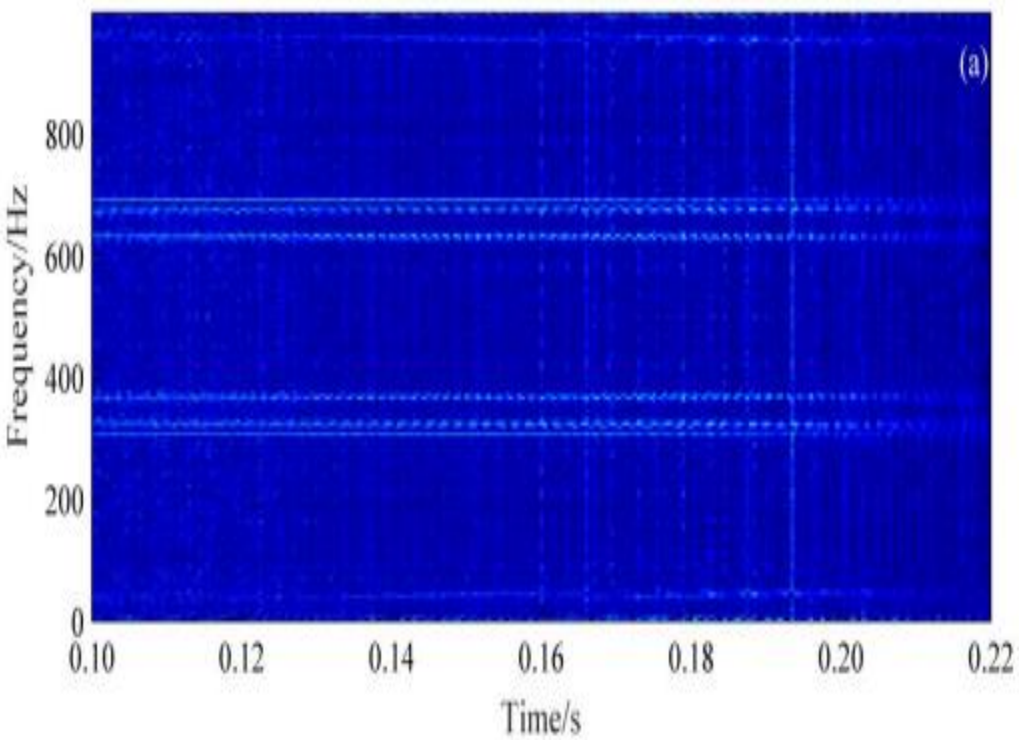
- 24^{th}-order wheel polygon
- OOR amplitude: 0.02 mm
- v = 300 km/h
- R = 460 mm
- $f = \frac{v}{\lambda} = \frac{nv}{2\pi R} = 24 * \frac{300(km/h)}{2\pi * 460(mm)} = 691.9\ Hz$

From Reference [1]

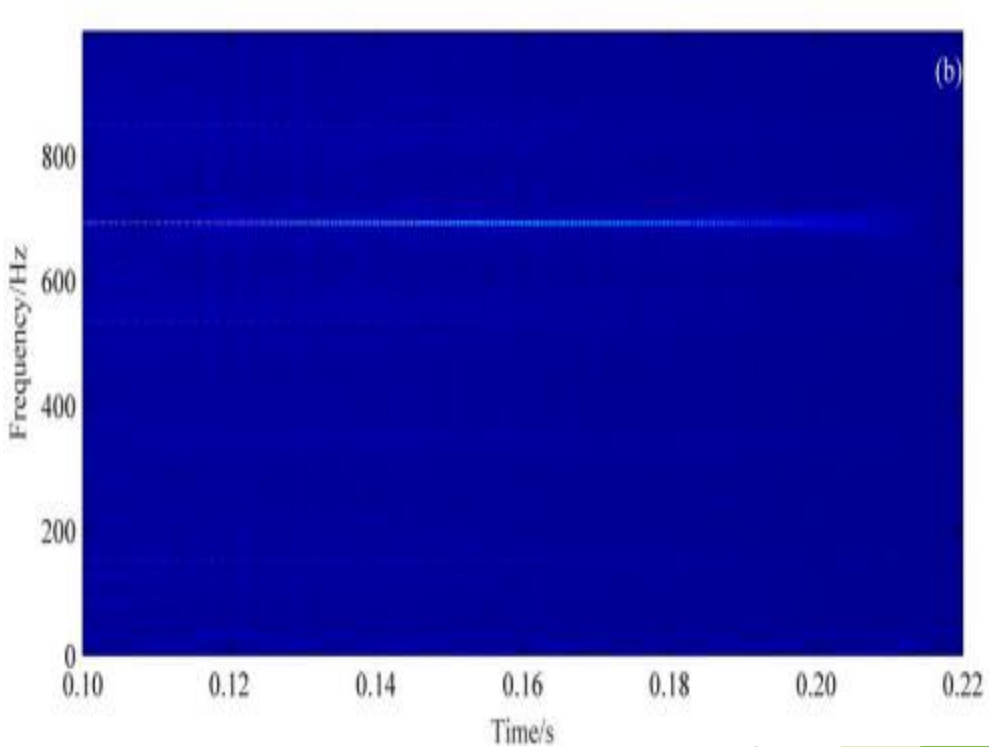
Results



f=691.9Hz



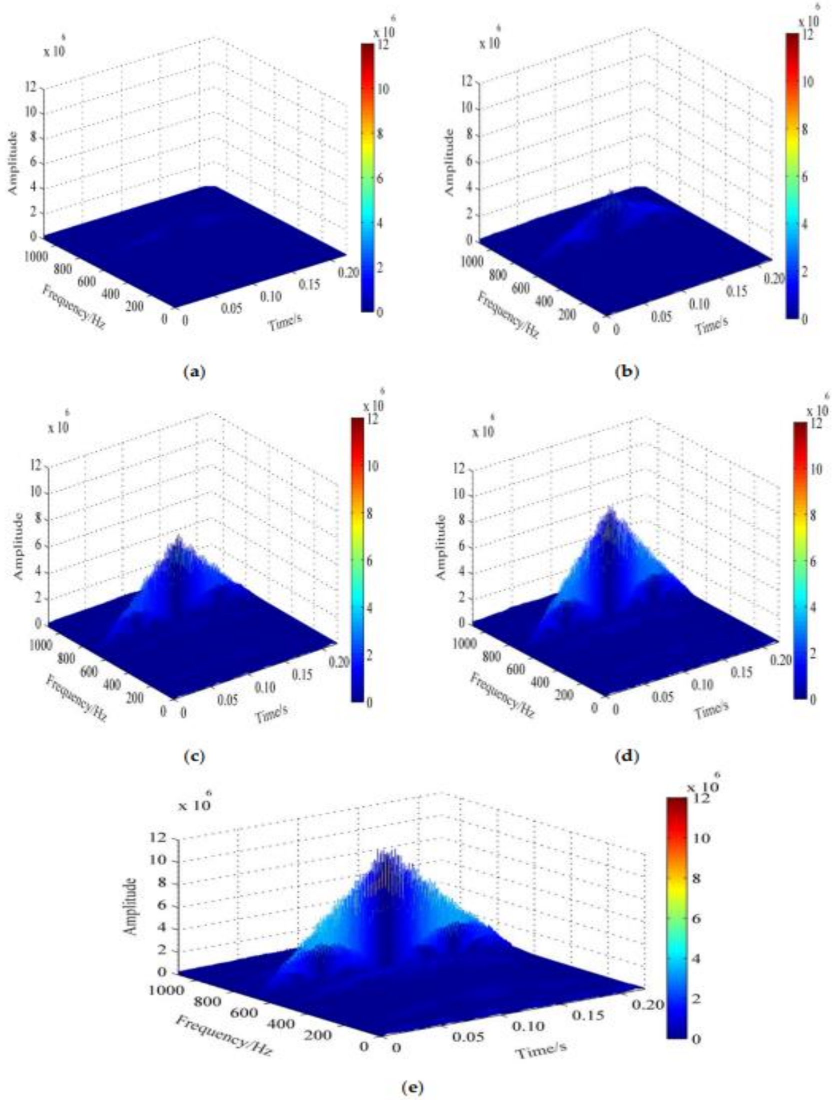
WVD

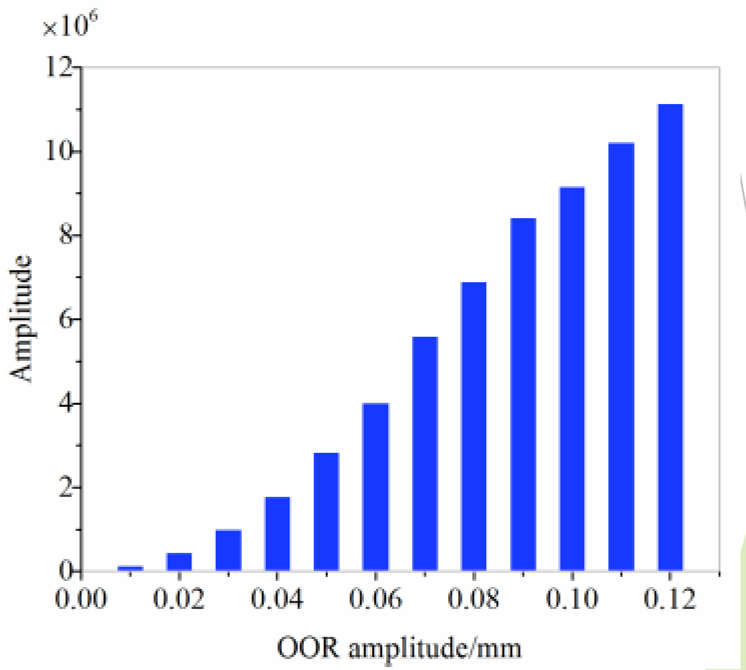


EEMD-WVD

From Reference [1]

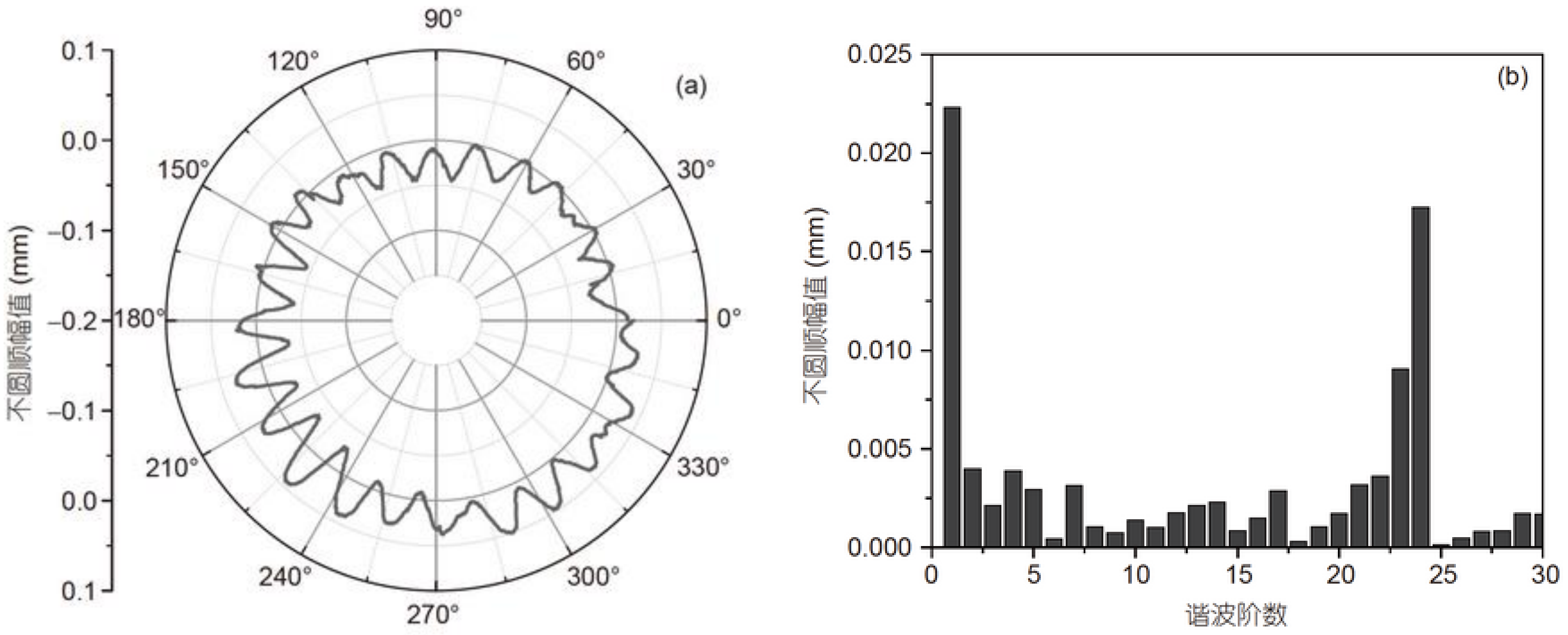
Results—change OOR amplitude





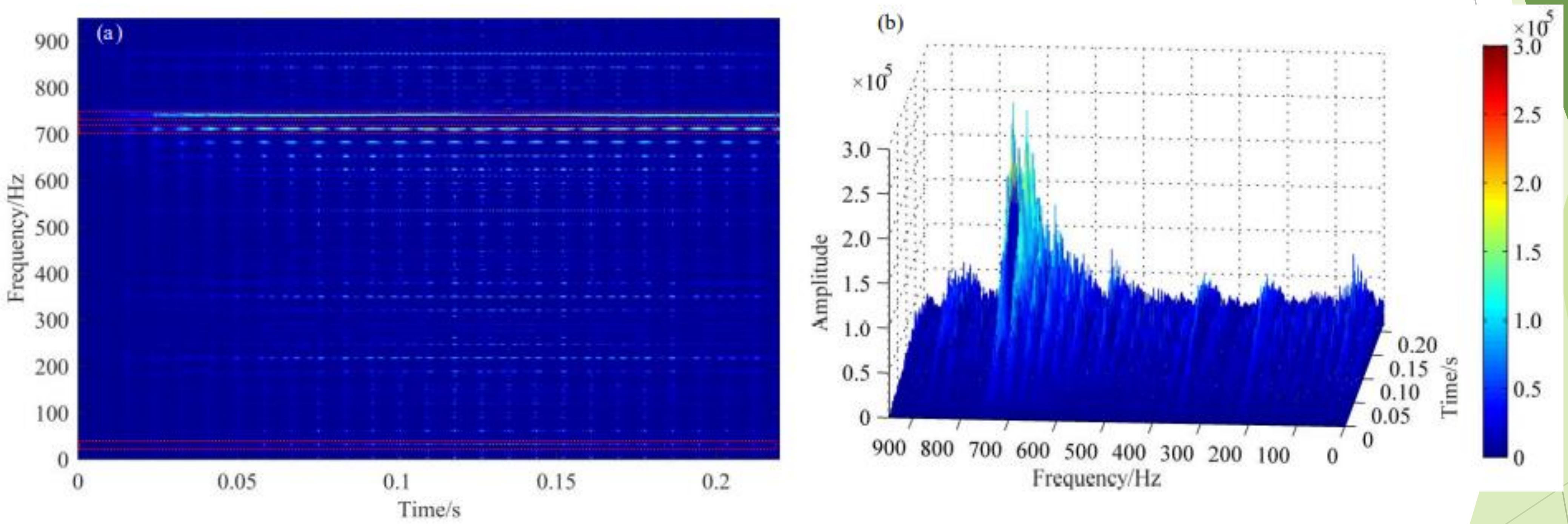
From Reference [1]

Results—real wheel



From Reference [8]

Results—real wheel



From Reference [1]

Reference

- [1] Song Y., Liang L., Du Y., Sun B. (2020). Railway Polygonized Wheel Detection Based on Numerical Time-Frequency Analysis of Axle-Box Acceleration. Applied Sciences, 10(5), 1613.
- [2] P. Salvador, V. Naranjo, R.Insa and P. Teixeira, “Axlebox accelerations: Their acquisition and time-frequency characterization for railway track monitoring purposes”, *Measurement*, vol. 82, pp. 301-302, Mar. 2016
- [3] Z. Li, M. Molodova, A. Nunez and R. Dollevoet, “Improvements in axle box acceleration measurements for the detection of light squats in railway infrastructure”, *IEEE Transactions on Industrial Electronics*, vol. 62, no. 7, pp.4387-4397, 2015.
- [4] J. C. Nielsen and A. Johansson, “Out-of-round railway wheels-a literature survey”, *Proc. Inst. Mech. Eng. F J. Rail Rapid Transit*, vol. 214, no. 2, pp. 79-91, 2000.
- [5] B. Liang, S. Iwnicki, A. Ball and A. E. Young, “Adaptive noise cancelling and time-frequency techniques for rail surface defect detection”, *Mechanical Systems and Signal Processing*, vol. 54, pp. 41-51, 2015.
- [6] 時頻分析與小波轉換 課程投影片
- [7] https://zh.m.wikipedia.org/zh-tw/%E5%B8%8C%E7%88%BE%E4%BC%AF%E7%89%B9-%E9%BB%83%E8%BD%89%E6%8F%9B
- [8] Chen, M., Zhai, W.M., et al., “Analysis of wheel-rail dynamic characteristics due to polygonal wheel passing through rail weld zone in high-speed railways”, *Sci. China Press* 64(25), 2573-2582, 2019. (in Chinese)