Wave shape function

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Outline

- Introduction
- Instantaneous Frequency and Wave-Shape Function (WSF)
- Adaptive Non-harmonic Model (ANHM) and Time-Varying Wave-Shape Functions
- Short Time Cepstral Transform (STCT) and De-shape algorithm
- Shape-Adaptive Mode Decomposition (SAMD)

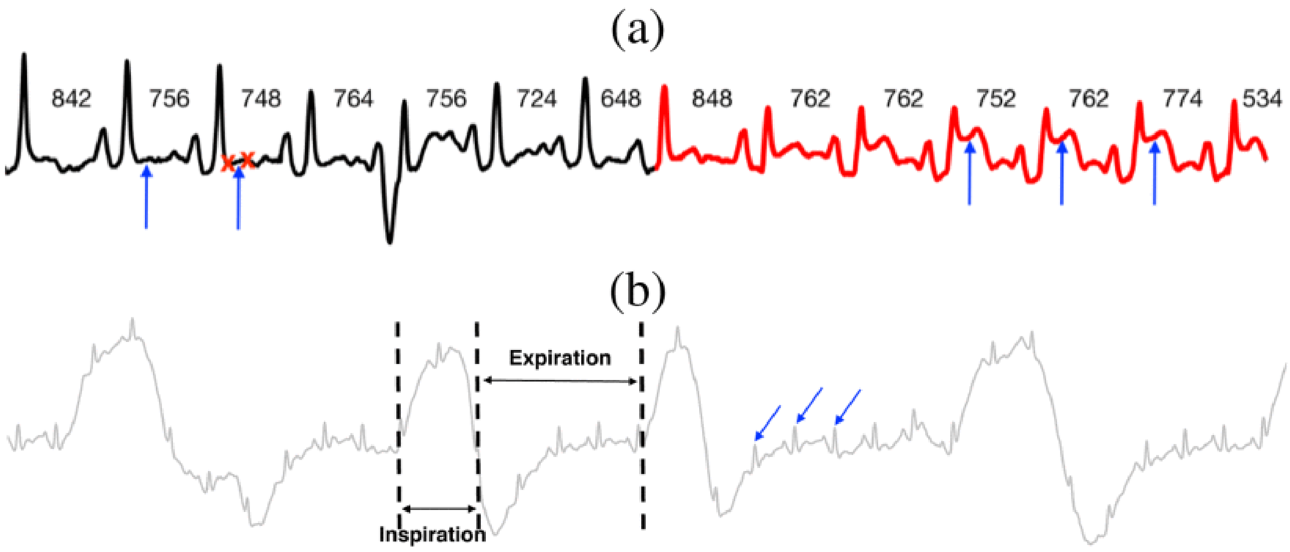
Introduction

Multi-component oscillating signals

- Time-varying frequencies
- Time-varying amplitudes
- Non-sinusoidal oscillation modes

Fields

- Biomedicine
- Mechanical systems
- Music signal



An illustration of two complicated non-sinusoidal oscillatory signals [1].

[1] M. A. Colominas and H. -T. Wu, "Decomposing Non-Stationary Signals With Time-Varying Wave-Shape Functions," in IEEE Transactions on Signal Processing, vol. 69, pp. 5094-5104, 2021

Adaptive Harmonic Model

For an simplest adaptive harmonic model [2]:

$$f(t) = \sum_{k=1}^{K} A_k(t)\cos\big(2\pi\phi_k(t)\big)$$

with $A_k(t), \phi_k'(t) > 0\ \forall t$.

- $A_k(t)$: amplitude modulation (slowly change) — $A_k(t) > 0, |A_k'(t)| \leq \epsilon\phi_k'(t)$
- $\phi_k(t)$: phase function (monotonic increasing) — $\phi_k'(t) > 0$
- $\phi_k'(t)$: instantaneous frequency (slowly change) — $|\phi_k''(t)| \leq \epsilon\phi_k'(t)$
- $A_k(t)\cos\big(2\pi\phi_k(t)\big)$: intrinsic mode type function

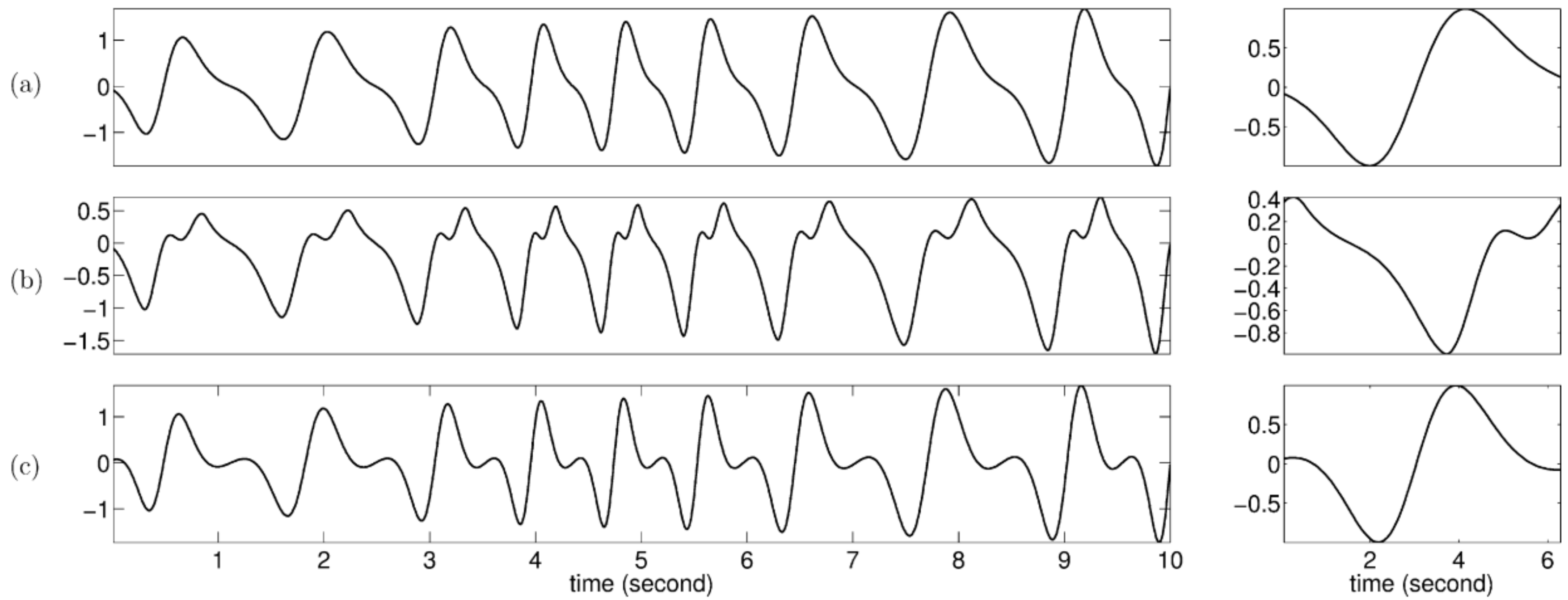
[2] I. Daubechies, J. Lu, and H.-T. Wu, “Synchrosqueezed wavelet transforms:An empirical mode decomposition-like tool,” Applied and Computational Harmonic Analysis, vol. 30, no. 2, pp. 243–261, 2011.

Wave-shape function (WSF)

The model for more real-life situation [3]:

$$f_{fix}(t) = A(t)s\big(2\pi\phi(t)\big)$$

where $s(t)$ is a real 1-periodic function with the unitary L^2 norm, that is for all t. The periodic signal $s(t)$ is called the wave-shape function.



Examples of WSF [3]

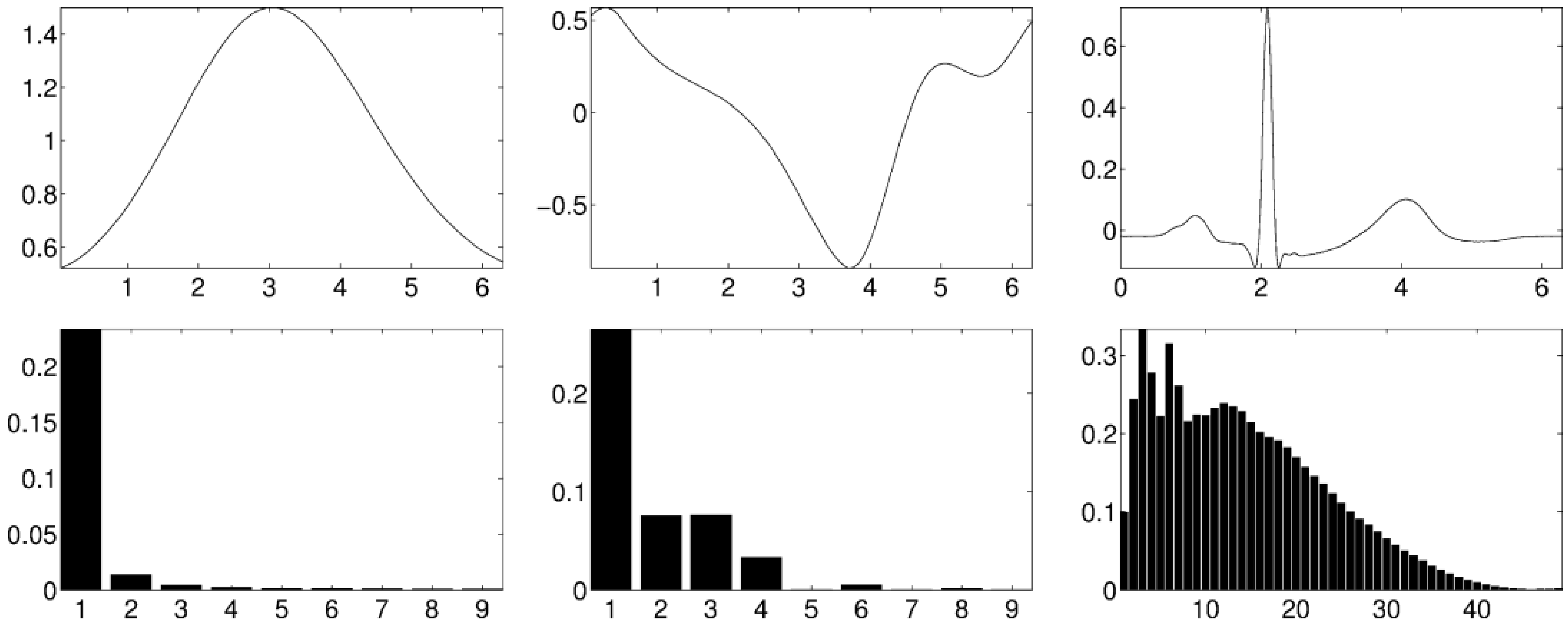
[3] H.-T. Wu, "Instantaneous frequency and wave shape functions (I)", Applied and Computational Harmonic Analysis, vol. 35, no. 2, pp. 181-199, 2013.

Wave-Shape Function (WSF)

Its Fourier series satisfies $s(t) = \sum_{l \in \mathbb{Z}} \hat{s}(l)\, e^{jl2\pi t}$, where $\hat{s}(l)$ are the Fourier coefficients.

S1: The fundamental component cannot be zero.

S2: Coefficients decay fast enough.



Three different shape functions [3]

[3] H.-T. Wu, "Instantaneous frequency and wave shape functions (I)", Appl. Comput. Harmon. Anal., vol. 35, no. 2, pp. 181-199, 2013.

Adaptive Non-Harmonic (ANH) Model

Replace cosine by an 1-periodic function (WSF) from the adaptive harmonic model.

The model was generalized to:

$$f(t) = \sum_{k=1}^{K} f_{fix,k}(t)$$

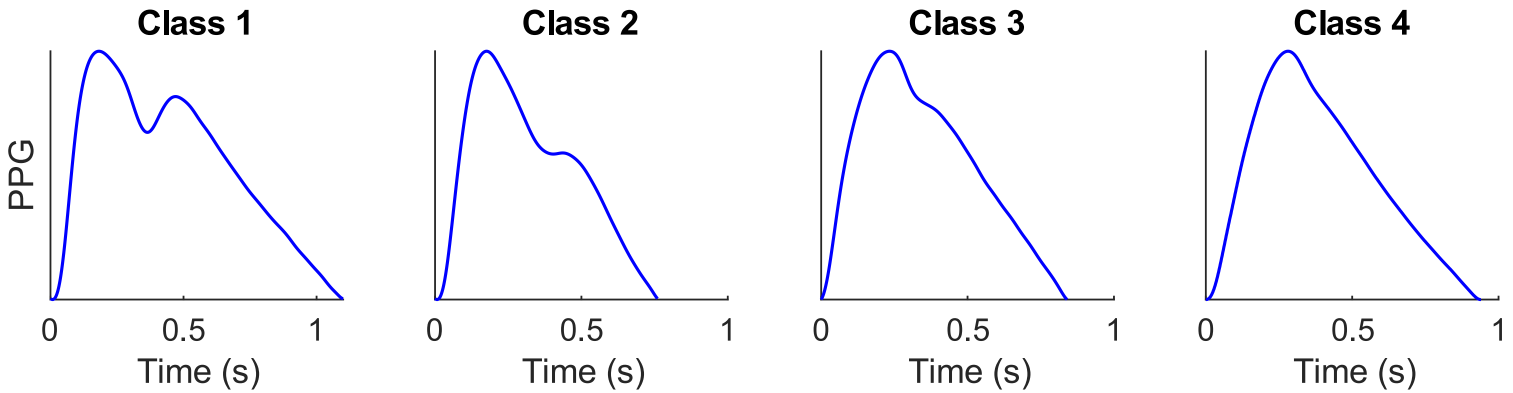
where $f_{fix,k}(t) = A_k(t) s_k\big(\phi_k(t)\big)$.

Since the WSF $s(t)$ is periodic, we can expand this by the Fourier series:

$$f_{fix,k}(t) = A_k(t) \sum_{l=1}^{\infty} \alpha_{k,l} \cos\big(2\pi l \phi_k(t) + \beta_{k,l}\big)$$

For real world case

- Electrocardiographic Signal
- Respiratory Signal
- Natural Vibration of Stiff Strings



Examples of the four classes of pulse wave shape proposed by Dawber et al. Reproduced form [4]

[4] Zanelli S, Eveilleau K, Charlton PH, Ammi M, Hallab M, El Yacoubi MA. Clustered photoplethysmogram pulse wave shapes and their associations with clinical data. Front Physiol. 2023 Oct 26;14:1176753.

Time-Varying Wave-Shape Functions

The model was generalized to:

$$f_{var}(t) = \sum_{l=1}^{\infty} A_l(t) \cos\big(2\pi\phi_l(t)\big)$$

- The IFs $\phi_l'(t)$ are not far from a multiple of the fundamental frequency $l\phi_1'(t)$.

The Adaptive non-Harmonic Model (ANHM) we consider in this paper satisfies

$$f(t) = \sum_{k=1}^{K} f_{var,k}(t)$$

where $f_{var,k}(t) = \sum_{l=1}^{\infty} A_{k,l}(t) \cos\left(2\pi\phi_{k,l}(t)\right)$.

- The fundamental IFs ($l = 1$) and their multiple components ($l \neq 1$) need to be separated.

De-shape Algorithm [5]

- For a chosen window function h, the definition of Short time Fourier transform (STFT):

$$V_f^h(t,\xi) = \int f(\tau)h(\tau - t)e^{j2\pi\xi(\tau-t)}d\tau$$

- The short time cepstral transform (STCT):

$$C_f^h(t,q) = \int \ln\left(\left|V_f^h(t,\xi)\right|\right) e^{j2\pi\xi q}d\xi$$

where q is called the quefrency.

- The inverse short time cepstral transform (iSTCT):

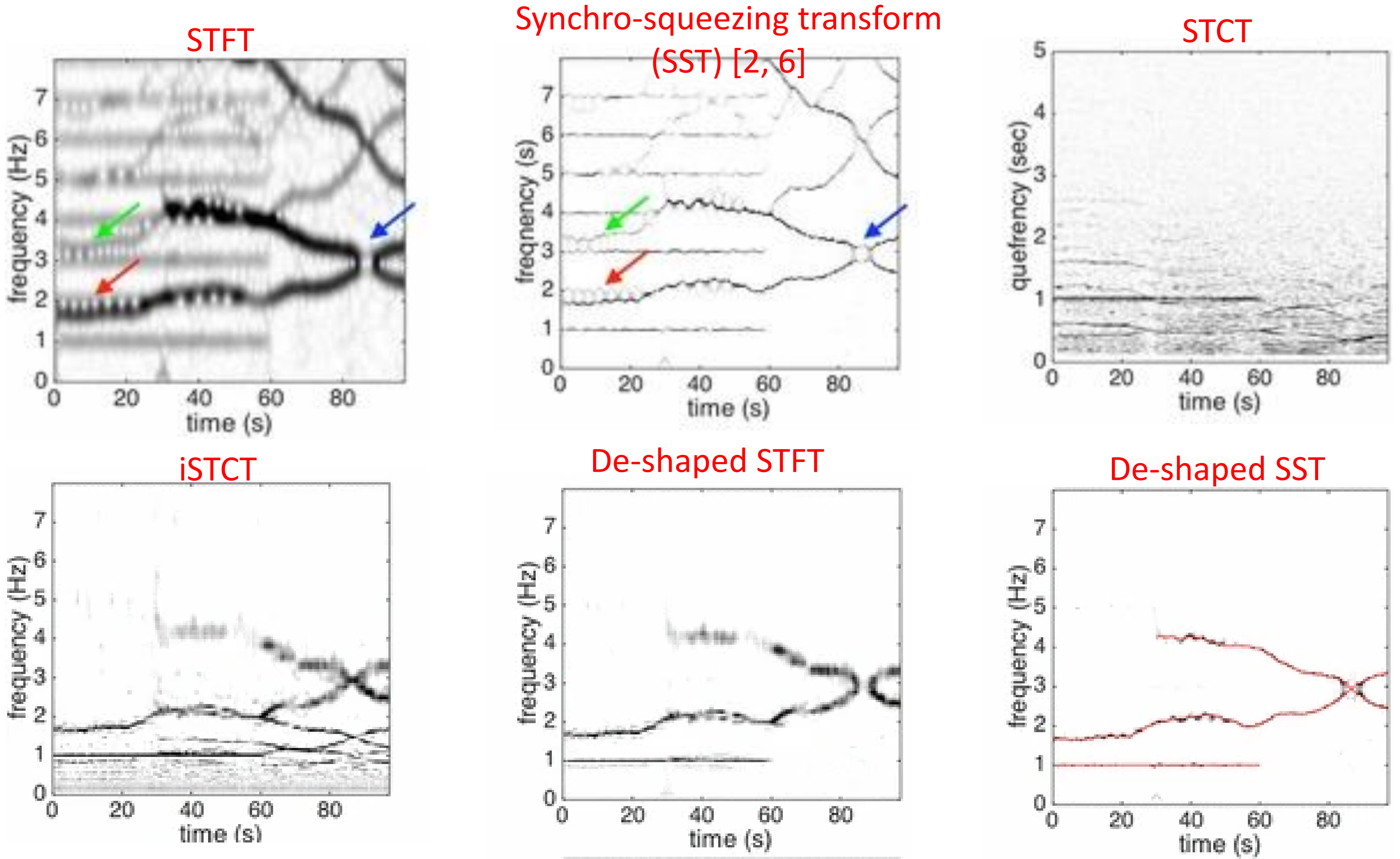
$$U_f^h(t,\xi) = C_f^h\left(t,\frac{1}{\xi}\right)$$

- The de-shape STFT:

$$W_f^h(t,\xi) = V_f^h(t,\xi)U_f^h(t,\xi)$$

[5] Lin, CY., Su, L. & Wu, HT. Wave-Shape Function Analysis. J Fourier Anal Appl 24, 451–505 (2018).

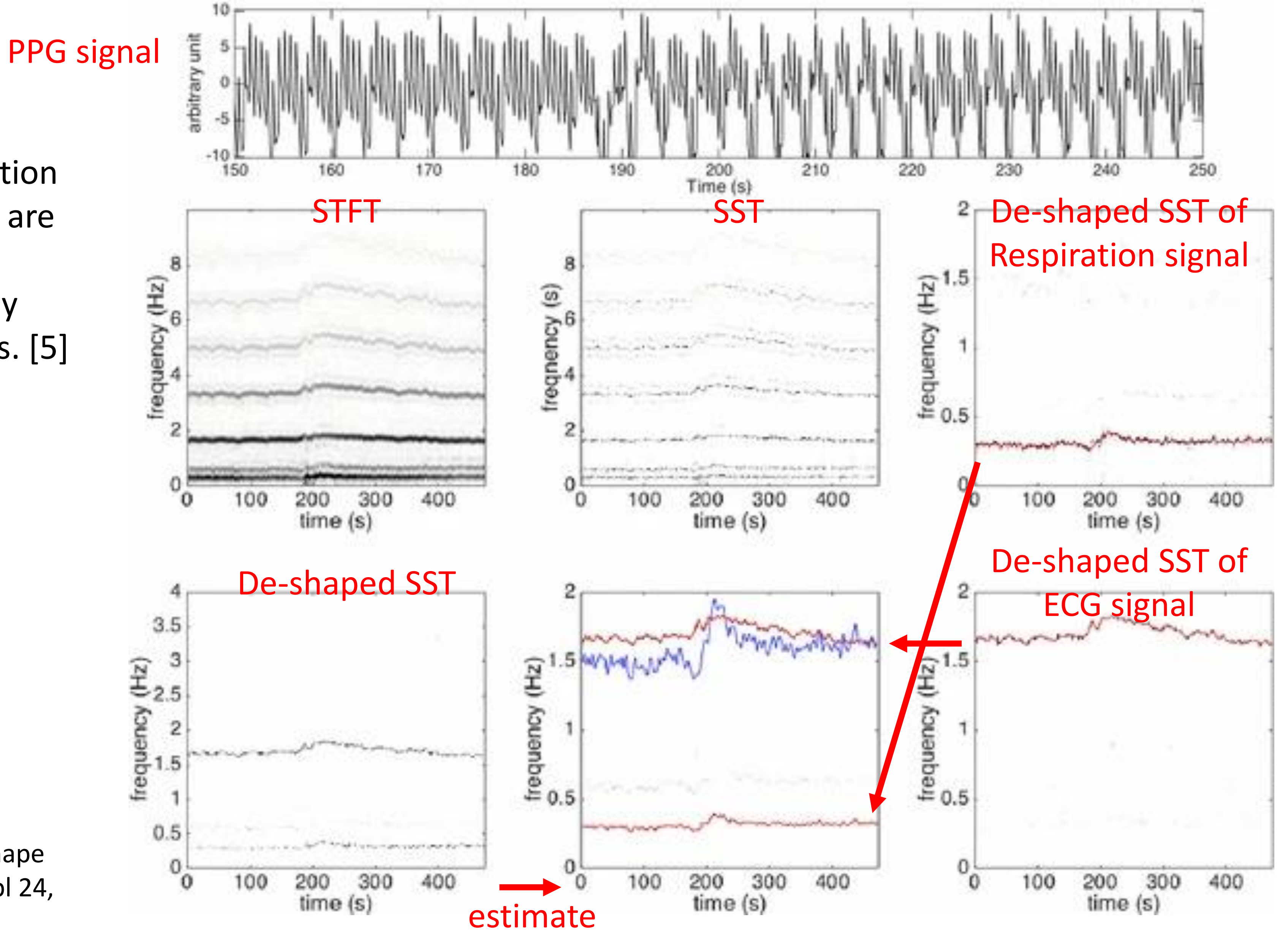
Three components simulated signal [5]



[5] Lin, CY., Su, L. & Wu, HT. Wave-Shape Function Analysis. J Fourier Anal Appl 24, 451–505 (2018).
[6] T. Oberlin, S. Meignen and V. Perrier, "The fourier-based synchrosqueezing transform," 2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Florence, Italy, 2014, pp. 315-319

photoplethysmography (PPG) signal

The PPG signal, the capnogram signal (respiration signal) and the ECG signal are simulateneously recorded from a subject without any motion at 300 Hz for 480 s. [5]



[5] Lin, CY., Su, L. & Wu, HT. Wave-Shape Function Analysis. J Fourier Anal Appl 24, 451–505 (2018).

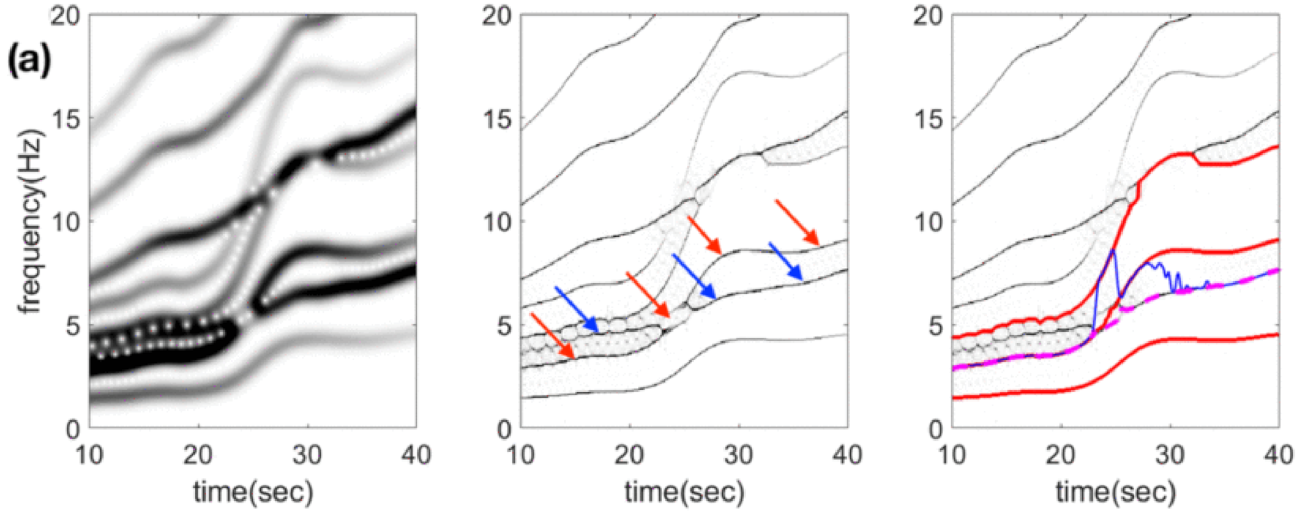
Ridge extraction (Instantaneous frequency estimation)

For the Time-Frequency representation $\mathbf{R}$, a frequently used method is to solve the following path optimization problem [7, 8]:

$$c^* = \arg\max_c \left(\sum_{j=1}^{N} \left|\widetilde{\mathbf{R}}\big(j, c(j)\big)\right| - \lambda_1 \sum_{j=1}^{N-1} |c(j+1) - c(j)|^2 \right)$$

where $\widetilde{\mathbf{R}}(l,q) = \log \frac{|\mathbf{R}(l,q)|}{\sum_{i,j}|\mathbf{R}(i,j)|}$ is a normalization of the matrix, $\lambda_1 > 0$ is the penalty term constraining the regularity of the fit.

[7] R. A. Carmona, W. L. Hwang and B. Torresani, "Characterization of signals by the ridges of their wavelet transforms," in IEEE Transactions on Signal Processing, vol. 45, no. 10, pp. 2586-2590, Oct. 1997
[8] Y. -W. Su, G. -R. Liu, Y. -C. Sheu and H. -T. Wu, "Ridge Detection for Nonstationary Multicomponent Signals With Time-Varying Wave-Shape Functions and its Applications," in IEEE Transactions on Signal Processing, vol. 72, pp. 4843-4854, 2024



Shape-Adaptive Mode Decomposition (SAMD)

- The optimization problem [1]:

$$\min_{c_{i,l},d_{i,l},\Phi_{i,l}} \left\| y(t) - \sum_{i=1}^{I} \tilde{A}_i(t) \sum_{l=1}^{D_i} \left(c_{i,l} \cos\left(\Phi_{i,l}(t)\right) + d_{i,l} \sin\left(\Phi_{i,l}(t)\right)\right)\right\|_2^2$$

- For more complex phases than those merely being integer multiples. Then, the evidently nonlinear regression problem

$$\min_{c_{i,l},d_{i,l},e_{i,l,k}} \left\| y(t) - \sum_{i=1}^{I} \tilde{A}_i(t) \sum_{l=1}^{D_i} \left(c_{i,l} \cos\left(\sum_{k=1}^{K} e_{i,l,k} \widetilde{\Phi}_{i,1}^{k}\right) + d_{i,l} \sin\left(\sum_{k=1}^{K} e_{i,l,k} \widetilde{\Phi}_{i,1}^{k}\right)\right)\right\|_2^2$$

- The Amplitude $\tilde{A}_i(t)$ and Phase $\widetilde{\Phi}_i(t) \approx 2\pi\tilde{\phi}_{i,1}(t)$ can be estimated by ridge detection with the SST.

$$\Phi_{i,l}(t) \approx \sum_{k=1}^{K} e_{i,l,k} \widetilde{\Phi}_{i1}^{k}$$

[1] M. A. Colominas and H. -T. Wu, "Decomposing Non-Stationary Signals With Time-Varying Wave-Shape Functions," in IEEE Transactions on Signal Processing, vol. 69, pp. 5094-5104, 2021

- Approximate $s_i(t)$ by its first D_i harmonics and solve the linear regression problem:

$$\min_{c_{i,l},d_{i,l}} \left\| y(t) - \sum_{i=1}^{I} \tilde{A}_i(t) \sum_{l=1}^{D_i} \left(c_{i,l} \cos\left(l\tilde{\Phi}_i(t)\right) + d_{i,l} \sin\left(l\tilde{\Phi}_i(t)\right)\right) \right\|_2^2$$

1. Ridge extraction for amplitudes and phases

2. Linear regression step for Fourier series parameters $c_{i,l}, d_{i,l}$

3. Non-linear regression step for parameters of high order polynomial phases $e_{i,l,k}$

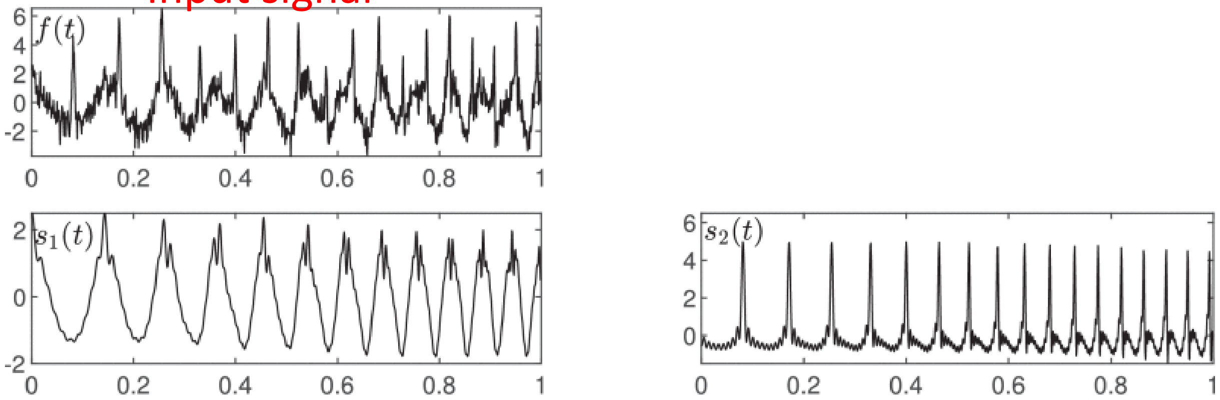
4. Synthesize the modes

Algorithm 1: Shape-Adaptive Mode Decomposition (SAMD).

1: **Input:** signal $y(t)$, and K (for phases estimations).
2: Estimate the amplitudes $\tilde{A}_i(t)$ and phases $\tilde{\phi}_{i1}(t)$ from second-order SST, ridge detection, and partial reconstruction (Section III-A), which also give an estimate of I.
3: Solve the linear regression problem to obtain the coefficients $\hat{c}_{i,\ell}$ and $\hat{d}_{i,\ell}$, which also give an estimate of parameters D_i (Section III-D1).
4: Solve the nonlinear regression problem from (17) using $\hat{c}_{i,\ell}$ and $\hat{d}_{i,\ell}$, and $e_{i\ell 1} = \ell$, $e_{i\ell k} = 0$ for $k = 2, \dots, K$, as initial values.
5: With the coefficients $\tilde{c}_{i,\ell}$, $\tilde{d}_{i,\ell}$ and $\tilde{e}_{i\ell k}$ synthesize the modes $f_{\mathrm{var},i}(t)$.
6: **Output:** modes $f_{\mathrm{var},i}(t)$, $i = 1, \dots, I$.

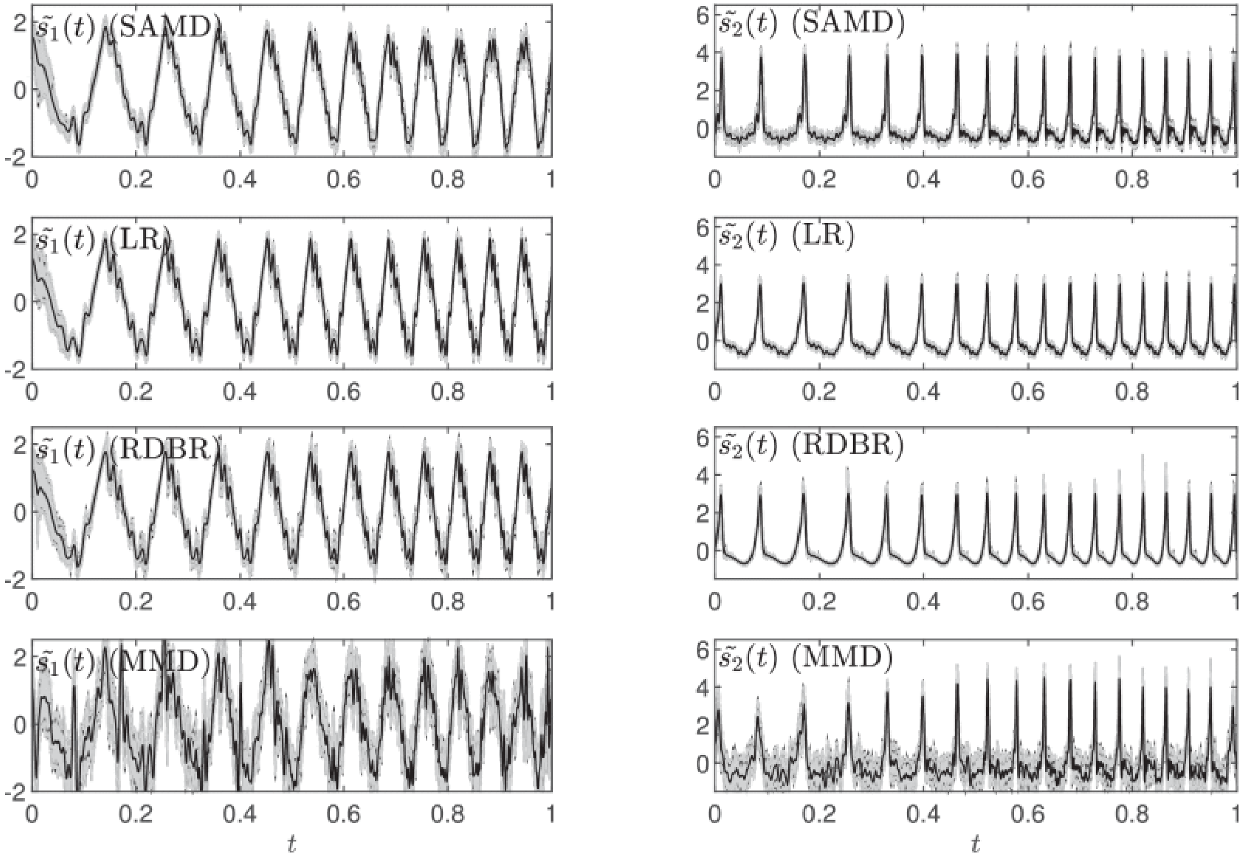
Experiments

Input signal



$s_1(t)$ $s_2(t)$

	Noiseless RMSE	Noisy (10 dB; 100 realizations) mean(RMSE)	std(RMSE)	mean time (s)
s_1 (SAMD)	0.206	0.288	0.017	43.210
s_2 (SAMD)	0.501	0.542	0.039	
s_1 (LR)	0.328	0.349	0.019	0.003
s_2 (LR)	0.627	0.638	0.043	
s_1 (RDBR)	0.330	0.355	0.021	14.245
s_2 (RDBR)	0.631	0.638	0.042	
s_1 (MMD*)	0.483	0.646	0.473	462.786
s_2 (MMD*)	0.483	0.611	0.063	



$s_1(t)$ $s_2(t)$

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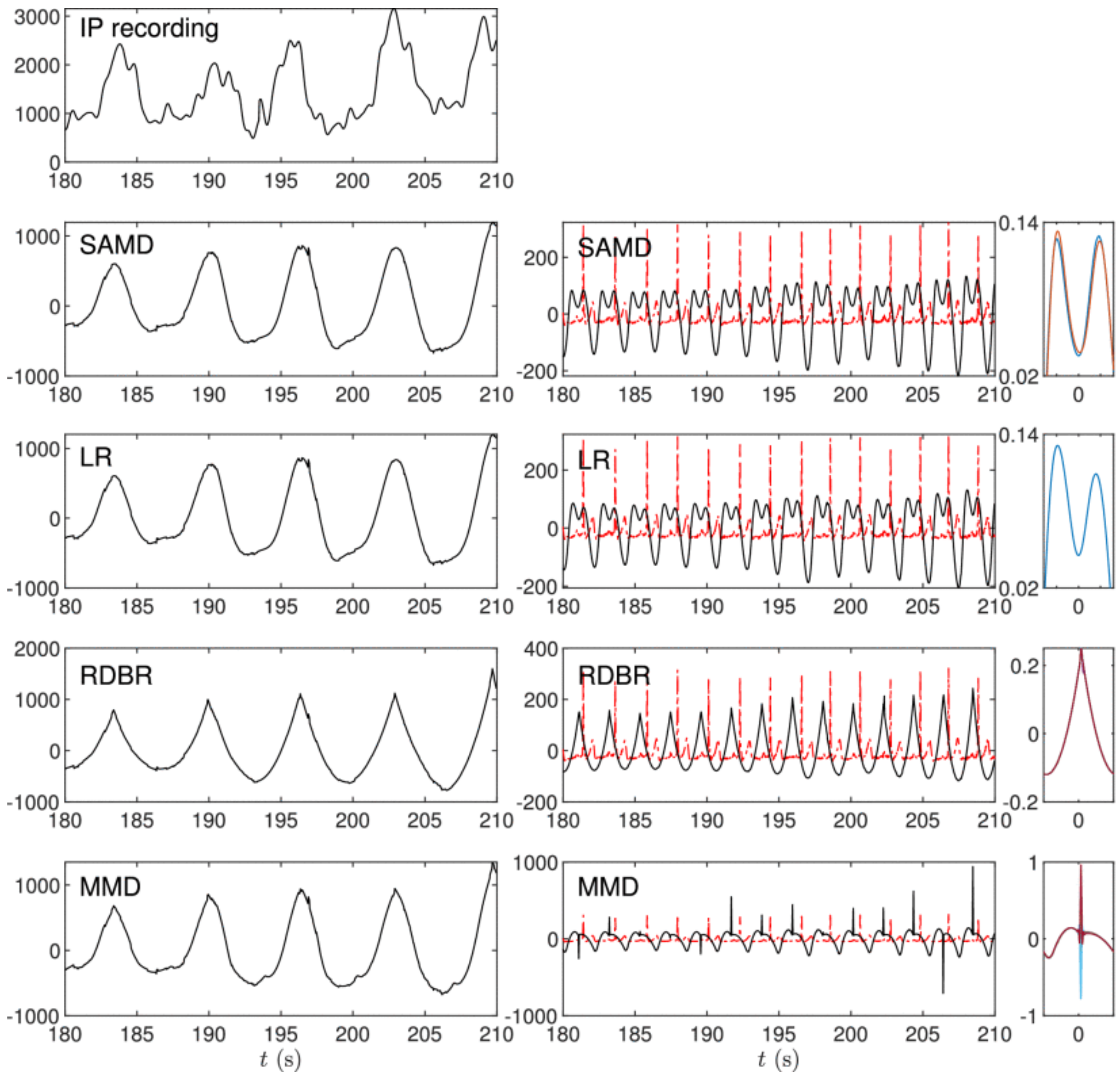
Experiments (impedance pneumography (IP))

An IP signal is usually composed of one respiratory component and one cardiac component, called the cardiogenic artifact.

For the 60-second segment, the computational times were 3.62 s, 0.007 s, 179.1 s, and 34.14 s for SAMD, LR, RDBR and MMD respectively.



[1] M. A. Colominas and H. -T. Wu, "Decomposing Non-Stationary Signals With Time-Varying Wave-Shape Functions," in IEEE Transactions on Signal Processing, vol. 69, pp. 5094-5104, 2021

Conclusion

- To handle oscillatory signals in the real world, a model is provided capturing oscillatory features, including IF, AM and time-varying wave-shape function.
- The idea of cepstrum introduce the STCT, de-shape STFT and de-shape SST.
- A novel nonlinear regression algorithm was proposed, SAMD, to decompose signals with time-varying WSFs.

References

[1] M. A. Colominas and H. -T. Wu, "Decomposing Non-Stationary Signals With Time-Varying Wave-Shape Functions," in IEEE Transactions on Signal Processing, vol. 69, pp. 5094-5104, 2021

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