2D Windowed Fourier Transform Concentration

鄭任傑

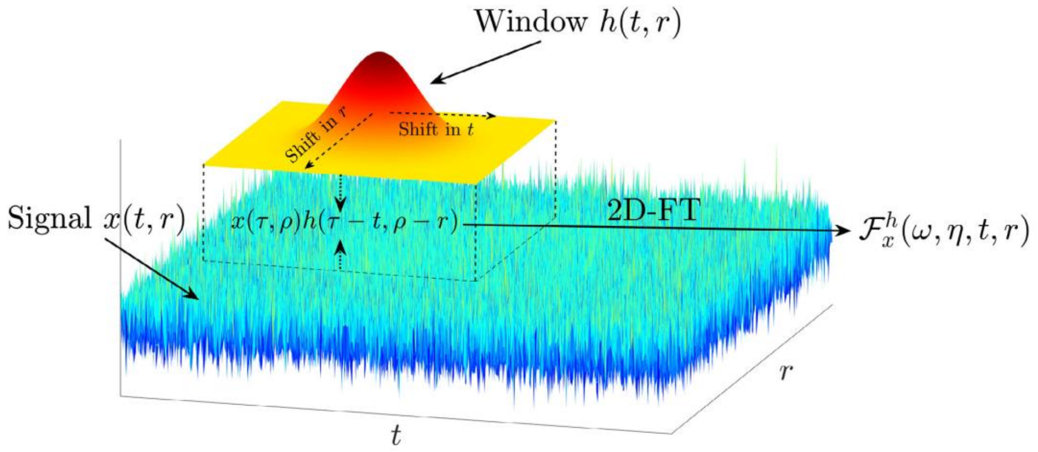
Outline

- Review windowed two-dimensional Fourier transform
- Estimator of windowed 2D Fourier transform
- Conclusion

The windowed 2D Fourier transform (W2D-FT)

The windowed 2D Fourier transform

$$F_x^h(\omega,\eta,t,r) = \int x(\tau,\rho)h(\tau-t,\rho-r)e^{-i(\omega\tau+\eta\rho)}d\tau\, d\rho$$



K. Abratkiewicz, “Windowed Two-Dimensional Fourier Transform Concentration and Its Application to ISAR Imaging,” *IEEE Transactions on Image Processing*, vol. 32, pp. 6260–6273, 2023.

Reassignment of W2D-FT

For the 2D signal model

$$x(t,r) = Ae^{i(\phi_x+\omega_x t)}e^{i(\psi_x+\eta_x r)}$$

The first order estimator of ω :

$$\hat{\omega}^{[1]}(\omega,\eta,t,r) = \omega - \Im\left\{\frac{F_x^{\partial_t h}(\omega,\eta,t,r)}{F_x^h(\omega,\eta,t,r)}\right\} = \omega_x$$

The first order estimator of η:

$$\hat{\eta}^{[1]}(\omega,\eta,t,r) = \eta - \Im\left\{\frac{F_x^{\partial_r h}(\omega,\eta,t,r)}{F_x^h(\omega,\eta,t,r)}\right\} = \eta_x$$

Reassignment

$$R(\omega,\eta,t,r) = \int\int F_x^h(\Omega,\xi,t,r)\delta\left(\Omega - \hat{\omega}^{[1]}(\Omega,\xi,t,r), \xi - \hat{\eta}^{[1]}(\Omega,\xi,t,r)\right)d\Omega d\xi$$

Reassignment of W2D-FT

Compute W2D-FT with window$\partial_t h, h$

Calculate the estimator $\hat{\omega}^{[1]}$, $\hat{\eta}^{[1]}$

Reassignment (moving energy)

Reassignment of W2D-FT

For the 2D signal model

$$x(t,r) = Ae^{i\left(\phi_x+\omega_x t+\frac{\alpha}{2}t^2\right)}e^{i\left(\psi_x+\eta_x r+\frac{\beta}{2}r^2\right)}$$

The second order estimator of ω :

$$\widehat{\omega}^{[2]}(\omega,\eta,t,r) = \omega + \Im\left\{\frac{F_x^{\partial_t Th}F_x^{Th} - F_x^{T^2h}F_x^{\partial_t h} + F_x^{Th}F_x^{h}}{\left(F_x^{Th}\right)^2 - F_x^{T^2h}F_x^{h}}\right\} = \omega_x + \alpha t$$

The second order estimator of η:

$$\hat{\eta}^{[2]}(\omega,\eta,t,r) = \eta - \Im\left\{\frac{F_x^{\partial_r Rh}F_x^{Rh} - F_x^{R^2h}F_x^{\partial_r h} + F_x^{Rh}F_x^{h}}{\left(F_x^{Rh}\right)^2 - F_x^{R^2h}F_x^{h}}\right\} = \eta_x + \beta r$$

Reassignment

$$R(\omega,\eta,t,r) = \int\int F_x^h(\Omega,\xi,t,r)\delta\left(\Omega - \widehat{\omega}(\Omega,\xi,t,r), \xi - \hat{\eta}(\Omega,\xi,t,r)\right) d\Omega d\xi$$

Reassignment of W2D-FT

Compute W2D-FT with several window $\partial_t Th, T^2h, Th, h$

↓

Calculate the $\widehat{\omega}^{[2]}$, $\hat{\eta}^{[2]}$

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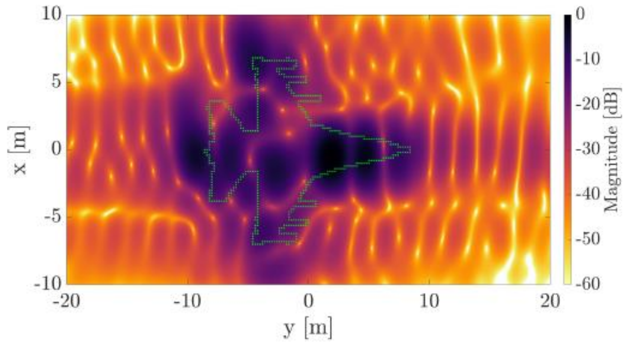
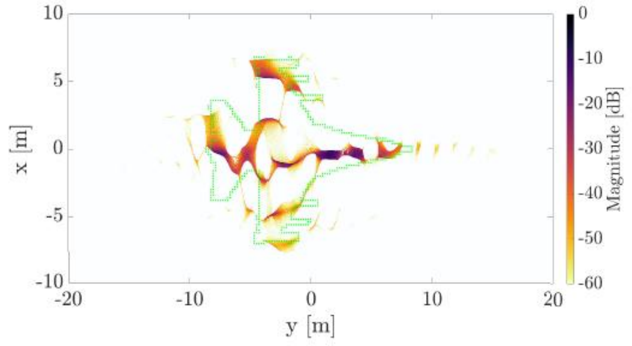
Reassignment (moving energy)

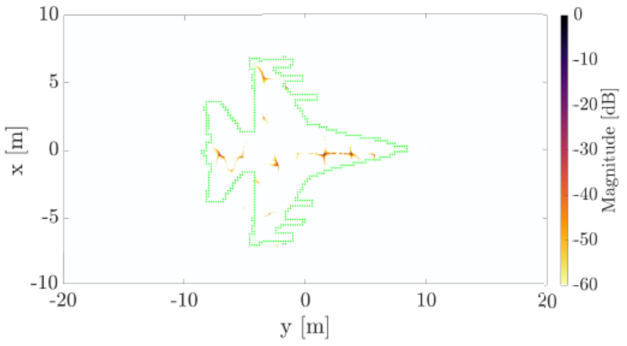
W 2DFT



1st order RM



2nd order RM



Entropy	1.2575	0.0489	0.0032

Smaller entropy represents more concentrated energy

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Conclusion

- Novel non-parametric technique 2D spectrum enhancement.
- Reassignment of 2D with 4 estimators.
- Improving the contrast and entropy.

Reference

1. K. Abratkiewicz, “Windowed Two-Dimensional Fourier Transform Concentration and Its Application to ISAR Imaging,” *IEEE Transactions on Image Processing*, vol. 32, pp. 6260–6273, 2023.