

Presentation: Reviews and talks on ICME 2016

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Introduction

ICME is a high-class conference on Multimedia.

Multimedia ?



Graphics

Speech

Vision

Display

Data
Mining

Video

Introduction

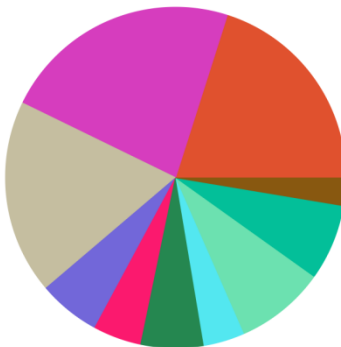
Top conference on multimedia and related

CVPR / ICCV, ECCV, ACM MM

Generally, ICME is suitable for MS(30%), PhD(65%) students to challenge. Recruitments from industrial are also a lot.

Introduction

Topics this year : (Video and Vision are the most popular in the trend)



Roadmap

1 Impressive Talks

- (1) Fei-Fei Li, Associate Professor, “A Quest for Visual Intelligence.” Computer Science Dept. Director, Stanford Artificial Intelligence Lab.
- (2) Mathias Wien, “High Efficiency Video Coding - Coding Tools and Specification: HEVC V3 and Coming Developments”

2 Interesting Works

- (1) Gene Cheung, Xian Ming Liu “Graph Signal Processing for Image Compression and Restoration,” Nii.
- (2) Xian Ming Liu, “Random Walk Graph Laplacian based Smoothness Prior for Soft Decoding of JPEG Images,” HIT.

A Quest for Visual Intelligence

Fei-Fei Li

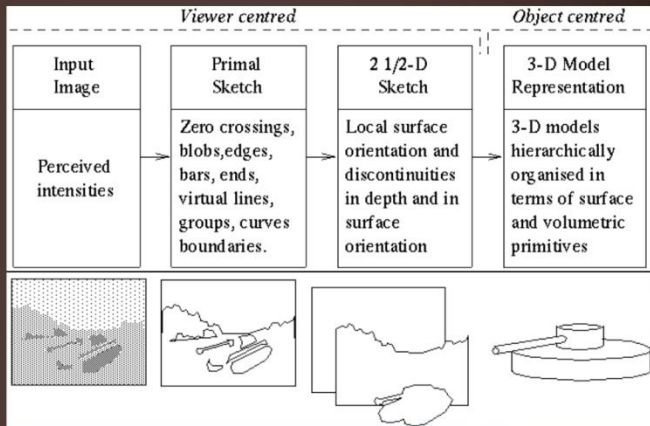
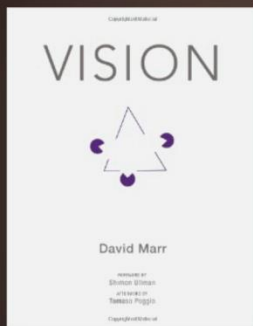
Associate Professor, Computer Science Dept.

Director, Stanford Artificial Intelligence Lab



Goal: total scene understanding

Fei-Fei Li



D. Marr, 1979

Fei-Fei Li

Computer
Vision



Machine
Learning

- Big progress in machine learning
 - SVM: Vapnik et al. 1995
 - AdaBoost: Freund & Schapire, 1995
 - Graphical models: Pearl 1988, Bishop 1995
 - MRF, CRF, MCMC, Gibbs, Variational, Non-parametric Bayes
 - Neural network: by many, 1950s onward



Fei-Fei Li

Object Recognition in 1990s and 2000s



[Harris and Stephens, '98]

[Lowe '99,'04]

[Mikolajczyk & Schmid '01,'04]

[Matas '02]

[Bay '06]

[Belongie & Malik '01]

[Berg & Malik '01]

[Oliva & Torralba '01, '06]

[Torralba '03]

[Fei-Fei '04]

[Dalal & Triggs '05]

[Lazebnik '06]

[Felzenswalb '08]

[Xiao, '10]

[Chatfield et '11]

Fei-Fei Li

Object Recognition in 1990s and 2000s

Pictorial Structure & Constellation Models



Felzenszwalb
et al. 2000
Fergus et al.
2003
Fei-Fei et al.
2003

Boosting



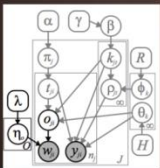
Viola & Jones
2001
Torralba et al.
2004

Bag of Words



Leung et al. 1999;
Sivic et al. 2003;
Grauman et al.
2005; Lazebnik et
al. 2006;
Fei-Fei et al. 2005

Non-parametric Bayes



Sudderth et
al. 2005
Li et al. 2007

Conditional Random Field



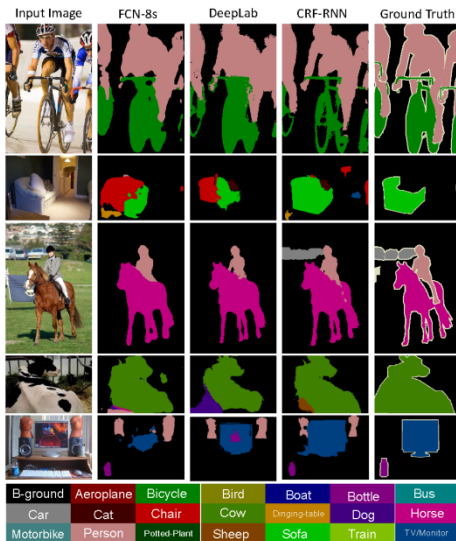
Kumar et al. 2003
Gould et al. 2009

And-Or Graphs

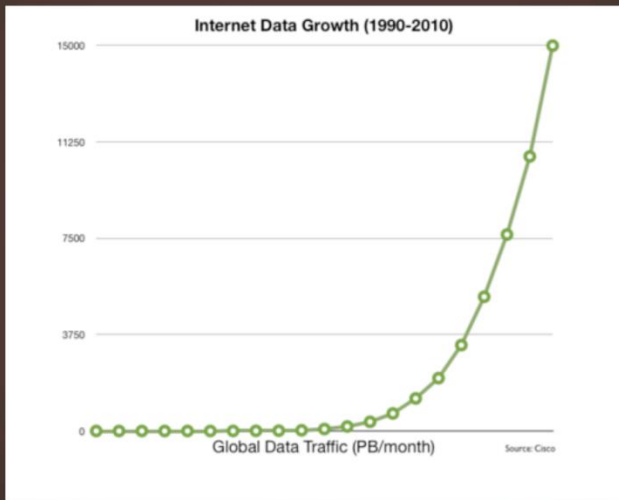


Chen et al. 2006
Zhu et al. 2007

Semantic Segmentation



Internet Data Growth (1990-2010)



IMAGENET

15,000,000 images in
22,000 categories

Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li & Li Fei-Fei, *CVPR*, 2009

Fei-Fei Li

Number of Labeled Images

SUN, **131K**
[Xiao et al. '10]

LabelMe, **37K**
[Russell et al. '07]

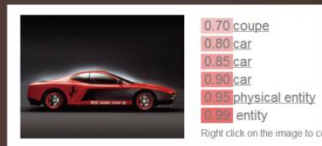
PASCAL VOC, **30K**
[Everingham et al. '06-'12]

Caltech101, **9K**
[Fei-Fei, Fergus, Perona, '03]

IMAGENET 15M

[Deng et al. '09]

Fei-Fei Li



J. Deng, J. Krause, A. Berg, & L. Fei-Fei, CVPR, 2012

Goal: total scene understanding

Sub-goal 2: Gist of a scene
(aka Image Captioning)

Fei-Fei Li

Some early efforts



Matching pictures and words

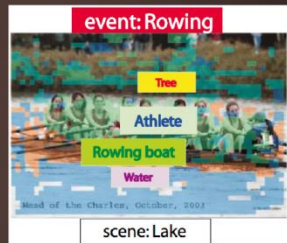
Barnard. et al. 2003

Duygulu, et al. 2002



Image parsing

Tu, Zhu, et al. 2003



What, where & who
Li & Fei-Fei, 2007

Model Outline & Core challenges



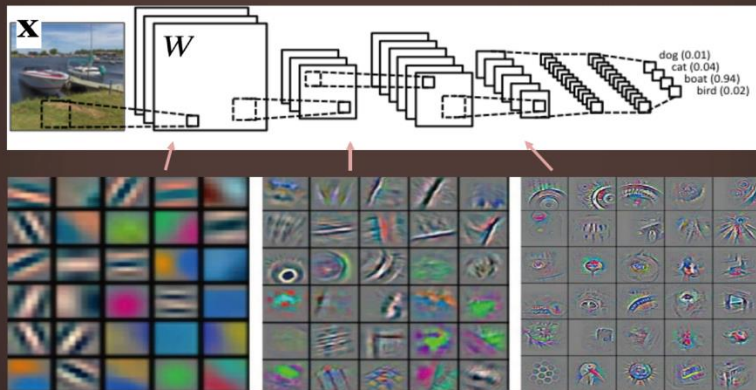
1. How do we
process images?

2. How do we
generate sentences

differentiable function

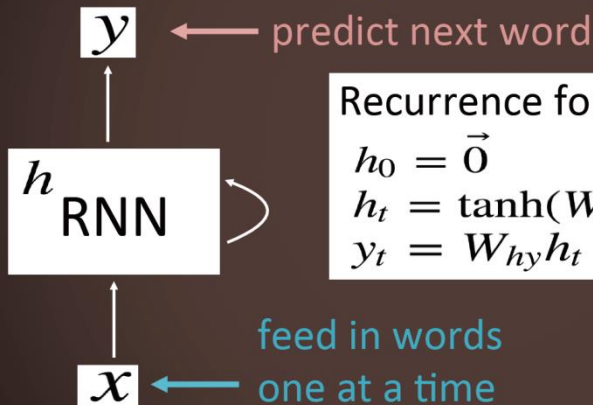
Fei-Fei Li

Convolutional Neural Network



[Zeiler & Fergus '13]

Recurrent Neural Network Language model



Recurrence formula:

$$h_0 = \vec{0}$$

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

$$y_t = W_{hy}h_t$$

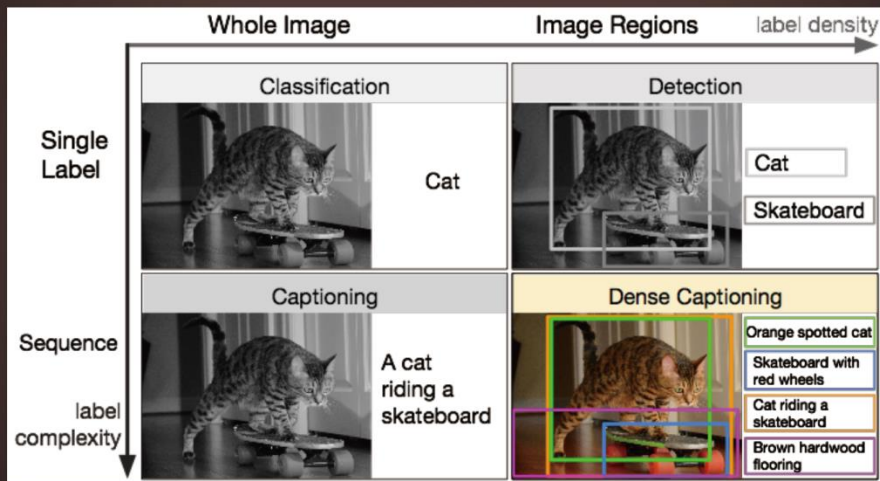
[Bengio et al., 2003]

[Mikolov et al., 2010]

[Sutskever et al., 2011]

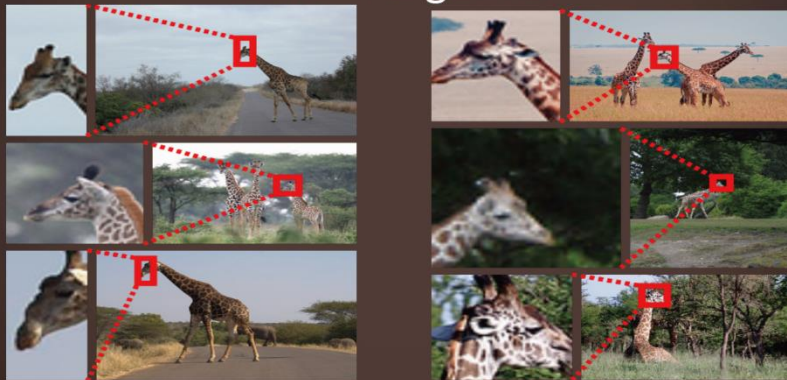
[Graves et al., 2013]

Dense Captioning



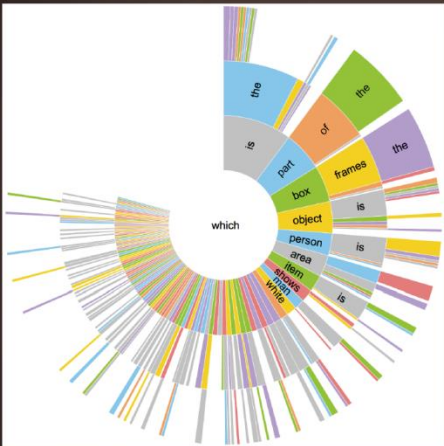
Fei-Fei Li

Finding regions given descriptions “head of a giraffe”



Justin Johnson, Andrej Karpathy & Li Fei-Fei, *CVPR*, 2016

Fei-Fei Li



Q: Which item is used to cut items?



Q: Which doughnut has multicolored sprinkles?



Q: Which man is wearing the red tie?



Q: Which pillow is farther from the window?



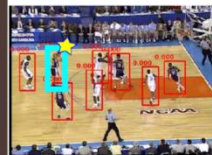
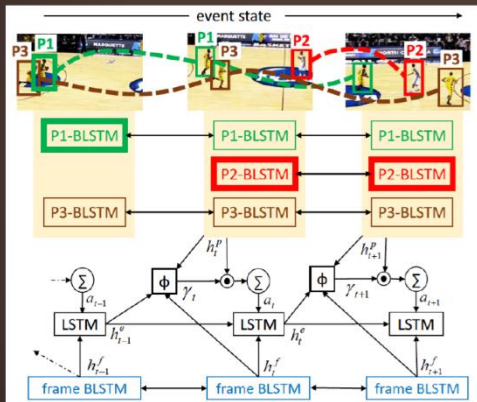
Q: Which step leads to the tub?



Q: Which is the small computer in the corner?

Yuke Zhu, Oliver Groth & Li Fei-Fei, CVPR 2016

Deep Learning for Action Detection

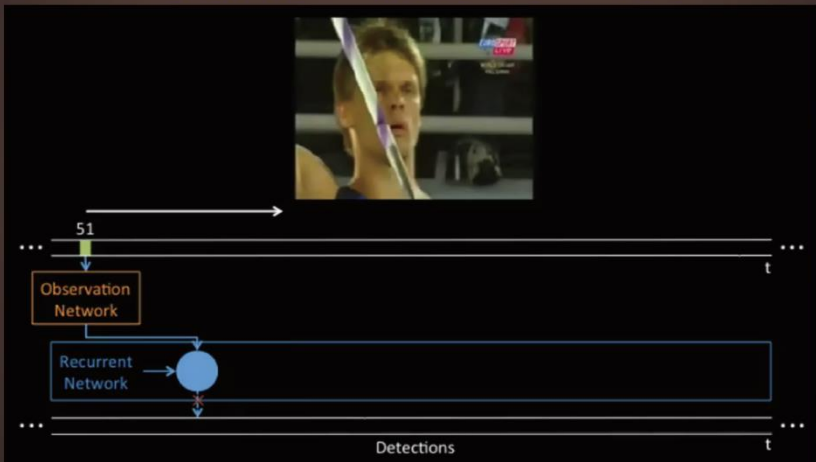


3-pointer success

Vignesh Ramanathan, Jon Huang, Alex Gorban, Kevin Murphy & Li Fei-Fei, *in submission*

Fei-Fei Li

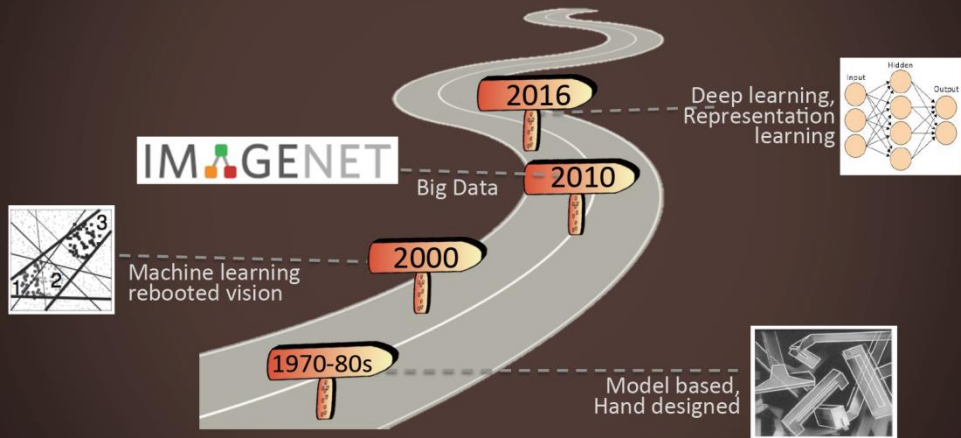
Ultra efficient event detection using a deep learning attention model



Serena Yeung, Olga Russakovsky, Greg Mori & Li Fei-Fei, CVPR, 2016

Fei-Fei Li

A journey of using knowledge, data, and learning

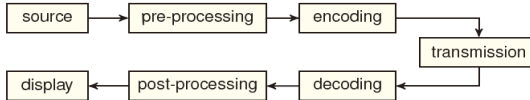


High Efficiency Video Coding – Coding Tools and Specification: HEVC V3 and Coming Developments

Mathias Wien

Institut für Nachrichtentechnik
RWTH Aachen University

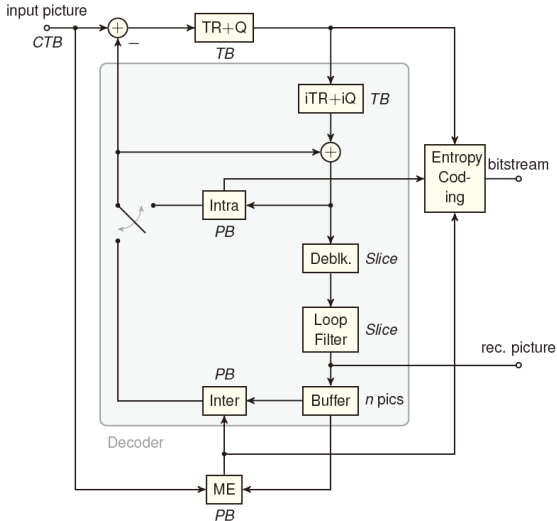
ICME2016



- Generalized overview of the processing chain
- Various realizations of the chain
 - Communication (e. g. video conferencing)
 - Broadcast (e. g. TV, streaming)
 - Storage (DVD, Blu-Ray, ...)
- Transcoding may be part of transmission

Mathias Wien

Hybrid Coding Scheme: Encoder

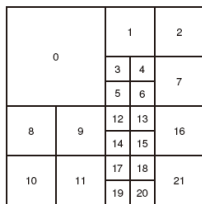


CTB – Coding Tree Block
ME – Motion Estimation
PB – Prediction Block
Q – Quantization
TB – Transform Block
TR – Transform

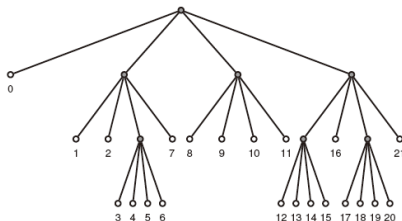
Mathias Wien

Coding Tree Blocks and Coding Blocks (CBs)

- Quadtree partitioning of CTB into CBs
- If picture size not integer multiple of CTB size:
Implicit CTB partitioning to meet picture size (must be multiple of 8×8 pixels)

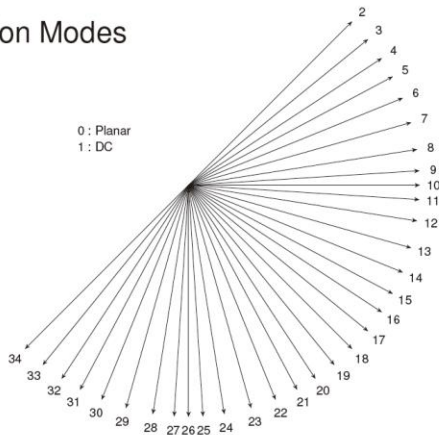


(a)



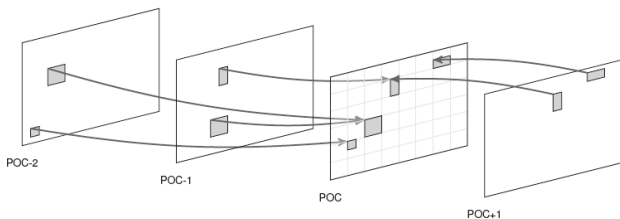
(b)

Intra Prediction Modes



- Planar prediction: mode 0
- DC intra prediction: mode 1
- Numbering from diagonal-up to diagonal-down
- Horizontal: mode 10, vertical: mode 26

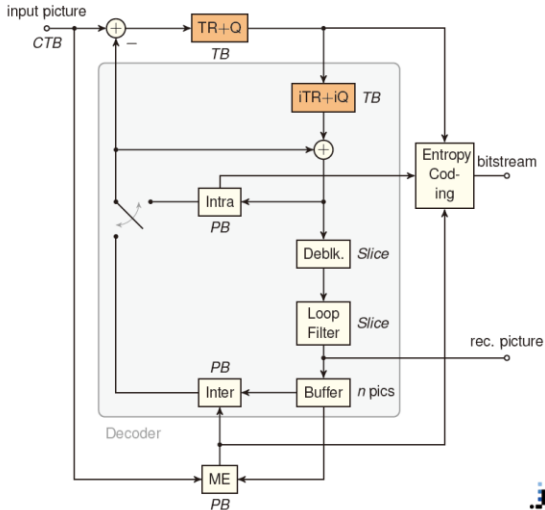
Motion Compensated Prediction



Prediction from reference picture lists

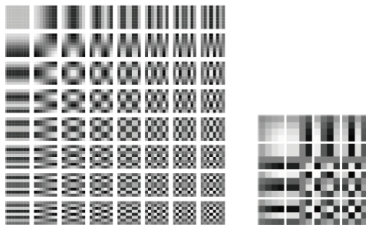
- Uni-prediction
 - P-slices only with List0, B-slices with List0 or List1
 - Minimum PB size 8×4 or 4×8
- Bi-prediction, only in B-slices
 - One predictor from List0, one predictor from List1
 - Minimum PB size 8×8

Residual Coding

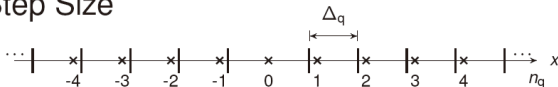


Core Transforms

- Transform block sizes 4×4 , 8×8 , 16×16 , and 32×32
- Integer approximations of the DCT-II transform matrix
- Additionally, integer approximation of the DST-VI transform matrix
- 'Single-norm' design per transform block size \rightarrow simple quantizer implementation
- Not all perfectly orthogonal, leakage below normalization threshold



Quantizer Step Size



- Quantizer step size Δ_q derived from quantization parameter QP
- Logarithmic relation of quantizer step sizes
- Double step size every 6 QP

$$\Delta_q(QP + 1) = \sqrt[6]{2} \cdot \Delta_q(QP)$$

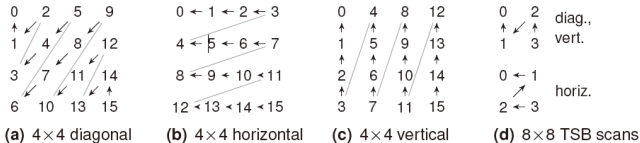
- Definition: $\Delta_q = 1$ for $QP = 4$, thereby

$$\Delta_{q,0} \in \left\{ 2^{-\frac{4}{6}}, 2^{-\frac{3}{6}}, 2^{-\frac{2}{6}}, 2^{-\frac{1}{6}}, 1, 2^{\frac{1}{6}} \right\}$$

- Quantizer step sizes for $QP > 5$

$$\Delta_q(QP) = \Delta_{q,0}(QP \bmod 6) \cdot 2^{\lfloor \frac{QP}{6} \rfloor}$$

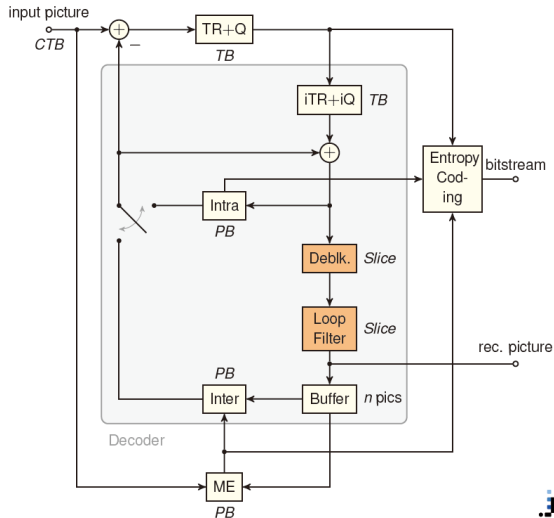
Transform Sub-Block Scanning



- Partitioning of transform block into 4×4 transform sub-blocks (TSBs)
- Scan direction selectable for 4×4 and 8×8 blocks, diagonal otherwise
- Scan direction in TSB depending on (intra) prediction mode
- Level distribution: 'trailing ones' expected towards higher frequencies
 - Scan used in *inverse* direction
 - Start with expected '1' values

Level: quantized transform coefficient level n_q

Loop Filtering



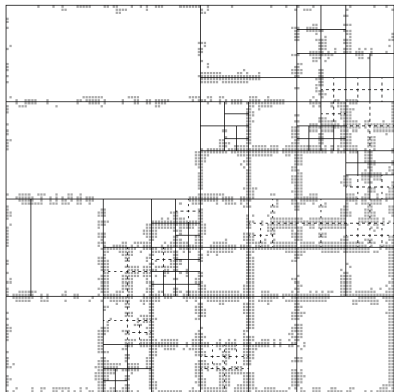
Deblocking Filter Operation

Deblocking filter operation

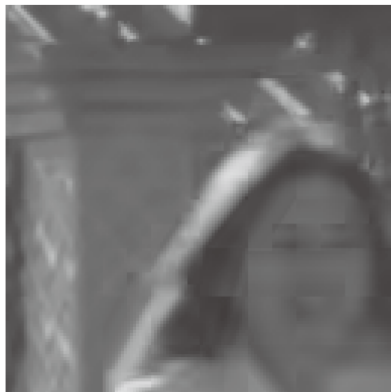
- Operation on a 4-sample edge basis
 - Luma: Deblocking if $b_s \geq 1$
 - Chroma: Deblocking if $b_s = 2$
- First vertical filtering, then horizontal filtering
- Independent operation on 8×8 block grid \rightarrow parallel processing!

Mathias Wien

Deblocking Filter Example

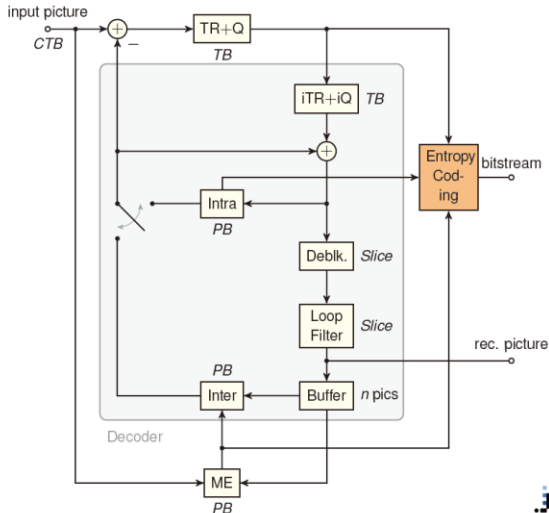


(c) Structure, deblocked samples

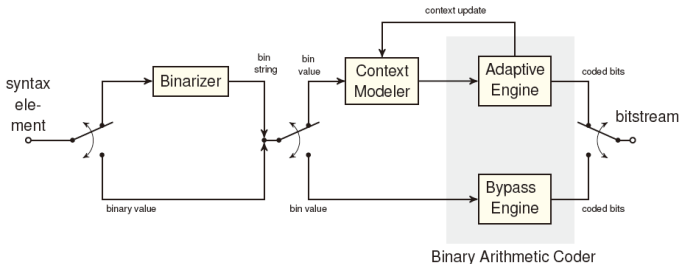


(d) Reconstruction without deblocking

Entropy Coding



Context-Based Adaptive Binary Arithmetic Coding – CABAC



- Binarization
- Context model selection
- Binary arithmetic coding
- Optimized binarization design, reduced number of non-bypass bins compared to H.264 | AVC

Extensions of HEVC

Range extensions (HEVC V.2/Ed. 2, 10/2014)

V.x=ITU-Version, Ed.y=ISO/ICE Edition

- Extended color formats (4:2:2, 4:4:4)
- Extended bit depth

Scalable extensions(HEVC V.2/Ed. 2, 10/2014)

- Simple, multiloop approach, no modifications on tool-level
- Supports spatial, SNR scalability

Multi-view (HEVC V.2/Ed. 2, 10/2014)

3D extensions (HEVC V.3/Ed. 3, 04/2015)

- Stereo / multi-view coding
- Multi-view with depth coding

Screen Content Coding (V.4/Ed. 3, 2016)

- Dedicated tools for this content type



JPEG PLENO

Clip slide



JPEG PLENO targets a standard framework for the representation and exchange of new imaging modalities such as **light-field**, **point-cloud** and **holographic imaging**.

SUMMARY AND NEXT STEPS FOR THE IMAGE COMPRESSION CHALLENGE

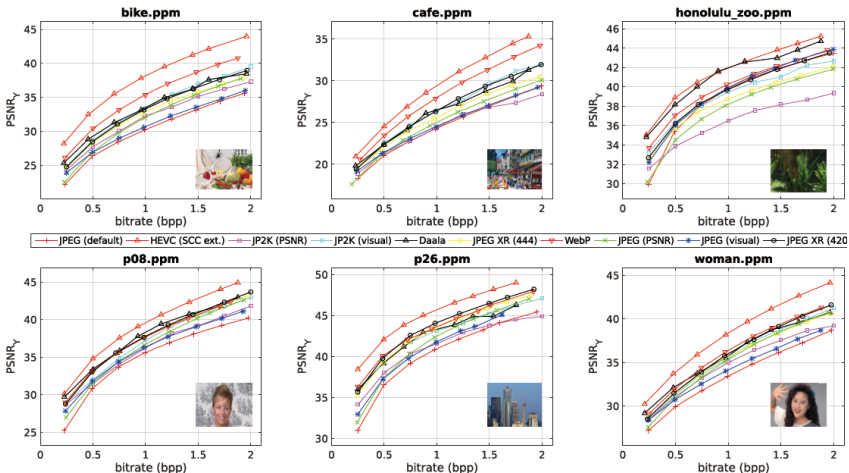
Evangelos Alexiou , Irene Viola,
Lukas Krasula, Thomas Richter, Tim Bruylants, Antonio Pinheiro, Karel Fliegel,
Martin Rerabek, Athanassios Skodras, Peter Schelkens and Touradj Ebrahimi

A contribution from Qualinet to JPEG call for information



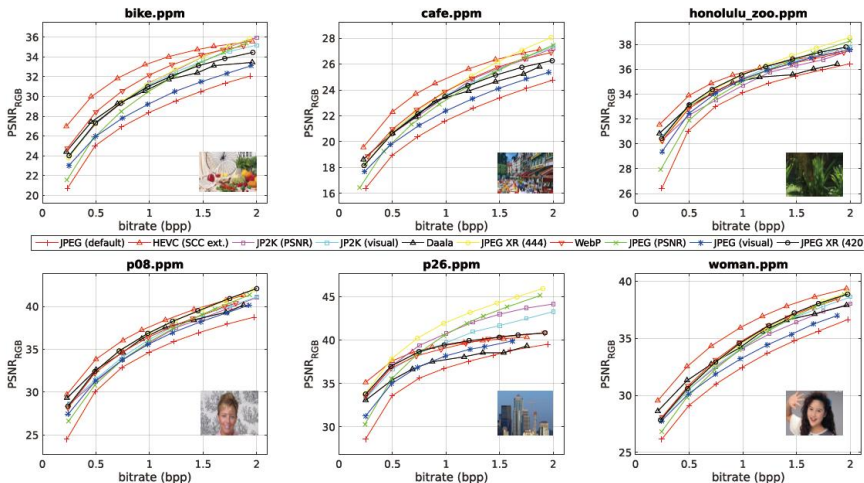
Objective evaluation: PSNR_Y results

18



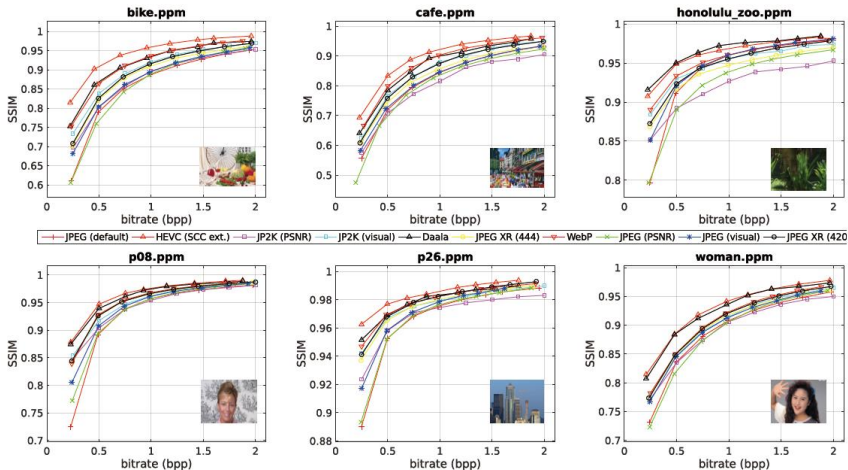
Objective evaluation: PSNR_{RGB} results

19



Objective evaluation: SSIM results

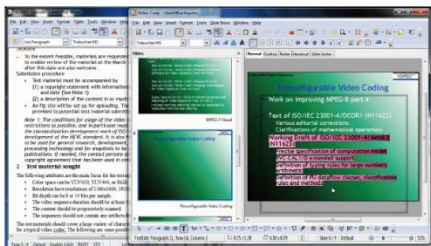
20



Mathias Wien

Screen Content Coding (SCC)

Characteristics of screen content sequences differ from camera captured video



Applications

- Gaming
- Remote desktops
- ...

Screen Content Coding Tools

Additional set of tools on top of RExt profile

- Intra block copy
- Palette mode coding
- Adaptive cross-component transformation
- Adaptive motion vector resolution

Gene Cheung

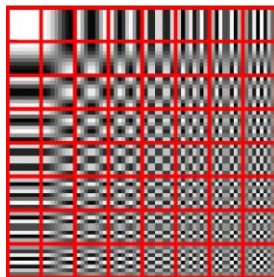
- (1) Gene Cheung, Xian Ming Liu “Graph Signal Processing for Image Compression and Restoration,” Nii.

Traditional signal property:

1. Discrete
2. Smooth
3. Band-limited frequency

Ex. DCT (Approx. of KLT)

$$X_k = \sum_{n=0}^{N-1} x_n \cos\left(\frac{\pi}{N}\left(n + \frac{1}{2}\right)k\right)$$

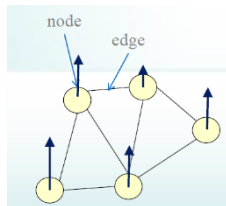
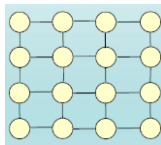


$$a = \Psi x$$

Dictionary representation inducing sparse coding

Graph Signal : Signal live on graph

Ex. Images: 2D - grid



Research interest:

1. Sampling : how to efficiently acquire signal from graph
2. Representation : Given the graph signal, how to compactly represent it.
3. Signal Restoration : given the partial or noisy signal, how to recover the structure.

Graph Fourier Transform (GFT)

Graph Laplacian:

- Adjacency Matrix A :** entry $A_{i,j}$ has *non-negative* edge weight $w_{i,j}$ connecting nodes i and j .
- Degree Matrix D :** diagonal matrix w/ entry $D_{i,i}$ being sum of column entries in row i of A .

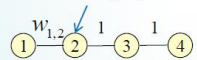
$$D_{i,i} = \sum_j A_{i,j}$$

- Combinatorial Graph Laplacian L :** $L = D - A$
 - L is *symmetric* (graph undirected).
 - L is a *high-pass* filter.
 - L is related to *2nd derivative*.

$$L_{3,:} x = -x_2 + 2x_3 - x_4$$

$$f''(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

undirected graph



$$A = \begin{bmatrix} 0 & w_{1,2} & 0 & 0 \\ w_{1,2} & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$


$$D = \begin{bmatrix} w_{1,2} & 0 & 0 & 0 \\ 0 & w_{1,2} + 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} w_{1,2} & -w_{1,2} & 0 & 0 \\ -w_{1,2} & w_{1,2} + 1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Graph Fourier Transform (GFT)


- Graph Fourier Transform** (GFT) is eigen-matrix of graph Laplacian L .

$$L u_i = \lambda_i u_i$$



- Recall classical **Fourier Transform**: of function f is inner-product with *complex exponentials*:

$$\hat{f}(\xi) = \langle f, e^{2\pi i \xi t} \rangle = \int f(t) e^{-2\pi i \xi t} dt$$




- Complex exponentials* are **eigen-functions** of 1D Laplace operator:

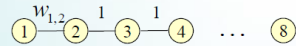
$$-\Delta(e^{2\pi i \xi t}) = \frac{\partial^2}{\partial t^2} e^{2\pi i \xi t} = (2\pi \xi)^2 e^{2\pi i \xi t}$$

- Analogously, GFT of graph-signal f is inner-product with **eigenvectors** of graph Laplacian L :

$$\hat{f}(\lambda_i) = \langle f, u_i \rangle = \sum_{n=1}^N f(n) u_i^*(n)$$



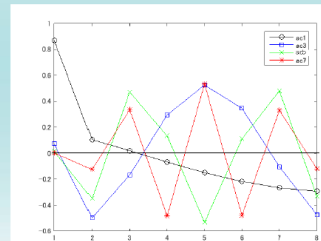
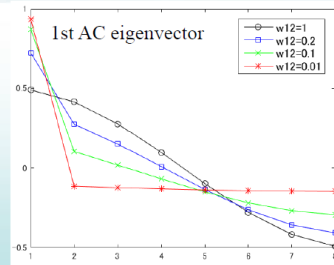
Graph Fourier Transform (GFT)



1. Sum of columns in $L = \mathbf{0} \rightarrow$ constant eigenvector assoc. with $\lambda_0 = 0$
2. Edge weights affect shapes of eigenvectors.
3. Eigenvalues (≥ 0) as *graph frequencies*.
 - Constant eigenvector is DC component.
 - # *zero-crossings* increases as λ increases.
4. GFT enables signal representation in graph frequency domain.

$\alpha = \Psi \mathbf{x}$

\nwarrow GFT
5. “Smoothness”, “band-limited” defined w.r.t. to graph frequencies.




ICME'16 Tutorial 07/11/2016

Variants of Graph Laplacians

- Graph Fourier Transform** (GFT) is eigen-matrix of graph Laplacian L .

$$L u_i = \lambda_i u_i$$



- Other definitions of graph Laplacians:

- Normalized** graph Laplacian:

$$L_n = D^{-1/2} L D^{-1/2} = I - D^{-1/2} A D^{-1/2}$$

- Random walk** graph Laplacian:

$$L_{rw} = D^{-1} L = I - D^{-1} A$$

- Generalized** graph Laplacian [1]:

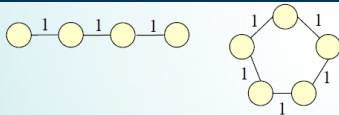
$$L_g = L + D^*$$

Characteristics:

- Normalized.
- Symmetric.
- No DC component.
- Normalized.
- Asymmetric.
- Eigenvectors not orthog.
- Symmetric.
- L plus self loops.
- Defaults to DST, ADST.

[1] Wei Hu, Gene Cheung, Antonio Ortega, "Intra-Prediction and Generalized Graph Fourier Transform for Image Coding," *IEEE*

Facts of Graph Laplacian & GFT



- $\mathbf{x}^T \mathbf{L} \mathbf{x}$ (graph Laplacian quadratic form) [2]) is one measure of variation in signal \rightarrow graph-signal smoothness prior.

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = \frac{1}{2} \sum_{i,j} w_{i,j} (x_i - x_j)^2 = \sum_i \lambda_i \alpha_i^2$$

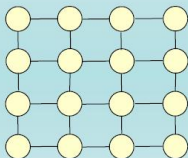
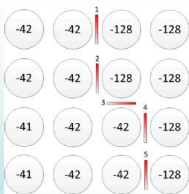
- Eigenvalues can be defined iteratively via Rayleigh quotient (Courant-Fischer Theorem):

$$\lambda_0 = \min_{f \in \mathbb{R}^N, \|f\|_2=1} \{f^T L f\}$$

$$\lambda_n = \min_{f \in \mathbb{R}^N, \|f\|_2=1, f \perp \text{span}\{u_0, \dots, u_{n-1}\}} \{f^T L f\} \quad n = 1, 2, \dots, N-1$$

- GFT defaults to **DCT** for un-weighted connected line.
- GFT defaults to **DFT** for un-weighted connected circle.

PWS Image Compression using GFT



- DCT are **fixed** basis. Can we do better?
- **Idea:** use **adaptive** GFT to improve sparsity [3].
 1. Assign edge weight 1 to adjacent pixel pairs.
 2. Assign edge weight 0 to sharp signal discontinuity.
 3. Compute GFT for transform coding, transmit coeff.
 4. Transmit bits (**contour**) to identify chosen GFT to decoder (**overhead of GFT**).

$$\alpha = \Psi x$$

← GFT

Shape-adaptive wavelets can also be done.

[3] G. Shen et al., "Edge-adaptive Transforms for Efficient Depth Map Coding," *IEEE Picture Coding Symposium*, Nagoya, Japan, December 2010.

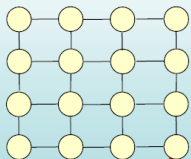
[4] M. Maitre et al., "Depth and depth-color Coding using Shape-adaptive Wavelets," *Journal of Visual Communication and Image Representation*, vol.21, July 2010, pp.513-522.

PWS Image Compression



Q: Why GFT leads to sparseness?

Ans 1: Capture statistical structure of signal in edge weights of graph.



- Adjacent pixel correlation 0 or 1 for **piecewise smooth** (PWS) signal.
- Can be shown GFT approximates KLT given **Gaussian Random Markov Field** (GRMF) model [5].

Ans 2: Avoid filtering across sharp edges.

- Low-freq GFT basis are PWS for PWS signals (discussed later).



a 4x4 block



GFT

$$\alpha_1 = \begin{bmatrix} 237 & 0 & 0 & 0 \\ 163 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

DCT

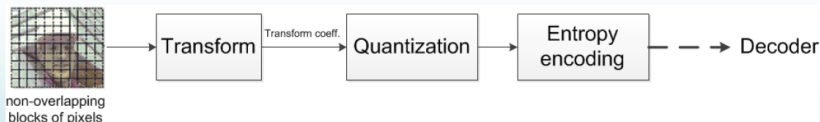
$$\alpha_2 = \begin{bmatrix} 285 & -29 & -5 & -4 \\ 16 & 1 & -16 & -4 \\ -5 & 3 & 5 & -7 \\ -1 & -4 & 1 & 9 \end{bmatrix}$$

filtering operation

$$\alpha = \Psi x$$

Graph Fourier Transform (GFT) for Block-based Image Coding

- Block-based Transform coding of images*



Two things to transmit for **adaptive transforms**:

- transform coefficients → the cost of transform representation
 - adaptive transform itself → the cost of transform description
-
- What's a *good* transform?
 - minimize the cost of **transform representation** & the cost of **transform description**

Transform Comparison

	Transform Representation	Transform Description
Karhunen-Loeve Transform (KLT)	“Sparsest” signal representation given available data / statistical model	Can be expensive (if poorly structured)
Discrete Cosine Transform (DCT)	<i>non-sparse signal representation</i> across sharp boundaries	little (fixed transform)
Graph Fourier Transform (GFT)	minimizes the total rate of signal’s transform representation & transform description	

Search for Optimal GFT

- Rate-distortion performance: $D + \lambda R$

D: distortion R: bit-rate

- **Assumption:** high bit rate, uniform quantization

Distortion does not change when considering different transforms! [6]

Consider Rate only!

- For a given image block $\mathbf{x} \in \mathbb{R}^N$ under fixed uniform quantization at high rate, the **optimal** GFT is the one that minimizes the total rate:

$$\min_{\mathbf{W}} R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W})$$

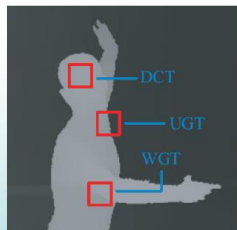
Rate of transform coefficient vector α

Rate of transform description T

MR-GFT: Definition of the Search Space for Graph Fourier Transforms

$$\min_{\mathbf{W}} R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W})$$

- In general, weights could be any number in $[0,1]$
- **To limit the description cost** R_T
 - Restrict weights to a small discrete set $\mathcal{C} = \{1, 0, c\}$



- For ease of computation, divide the optimization into two sub-problems

1. **Weighted GFT** (WGFT): $\mathcal{C}_1 = \{1, c\}$

2. **Unweighted GFT** (UGFT): $\mathcal{C}_2 = \{1, 0\}$

Strong correlation only? Default to the DCT

WGFT

$$\begin{aligned} \min_{\mathbf{W}} \quad & R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W}) \\ \text{s.t.} \quad & W_{i,j} \in \{1, c\} \quad \forall i, j \in \mathcal{V} \end{aligned}$$

- Cost function of **transform coefficients**

$$\hat{R}_{\alpha}(\mathbf{x}, \mathbf{W}) \approx \mathbf{x}^T \mathbf{L} \mathbf{x} = \sum_{(i,j) \in \mathcal{E}} W_{ij} (x_i - x_j)^2 = \sum_k \lambda_k \alpha_k^2$$

GFT coeff
graph freq.

- Cost function of **transform description**

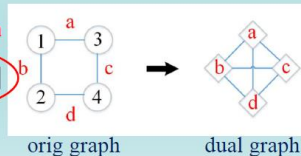
$$\hat{R}_T(\mathbf{W}) = \sum_{(e,s) \in \mathcal{E}^d} |W_e - W_s| + \sum_{e \in \mathcal{V}^d} \gamma \rho(1 - W_e)$$

costly if many weight changes code only non-1's

- Problem formulation for WGFT **deviation**

$$\min_{\mathbf{W}} \quad \rho \sum_{e \in \mathcal{V}^d} [W_e (x_{v_1(e)} - x_{v_2(e)})^2 + \gamma(1 - W_e)] + \sum_{(e,s) \in \mathcal{E}^d} |W_e - W_s|$$

$$\text{s.t.} \quad W_e \in \{1, c\} \quad \forall e \in \mathcal{V}^d.$$



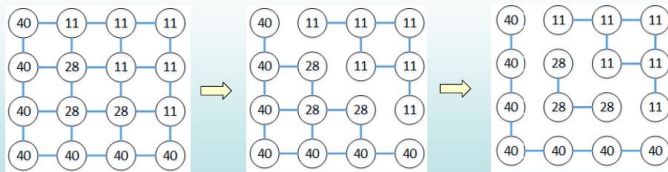
- Separation-Deviation** (SD) problem, solvable in polynomial time [8].

[8] D. S. Hochbaum, "An Efficient and Effective Tool for Image Segmentation, Total Variations and Regularization," *SSVM'11 Proceedings of the Third International Conference on Scale Space and Variational Methods in Computer Vision*, 2011, pp.338-349.

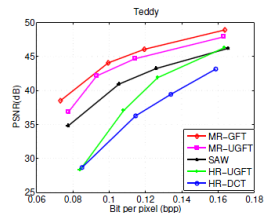
UGFT

$$\begin{aligned} \min_{\mathbf{W}} \quad & R_{\alpha}(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W}) \\ \text{s.t.} \quad & W_{i,j} \in \{1, 0\} \quad \forall i, j \in \mathcal{V} \end{aligned}$$

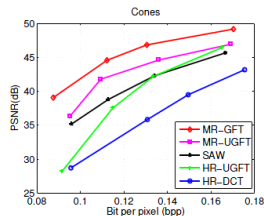
- A greedy algorithm



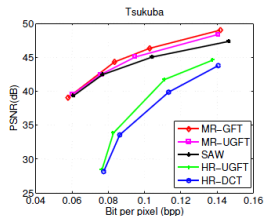
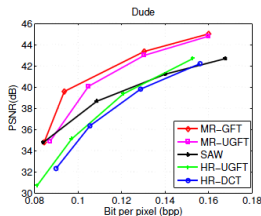
- Divide graph into disconnected sub-graphs via spectral clustering [9].
- Check objective function, further sub-divide if cost decreases.



(a)



(b)



Wei Hu, Gene Cheung, Antonio Ortega, Oscar Au, "Multiresolution Graph Fourier Transform for Compression of Piecewise Smooth Images," *IEEE Transactions on Image Processing*, vol.24, no.1, pp.419-433, January 2015.

Skipped part

1. PWS image coding : Generalized GFT with intra-coding of H.264
2. Lifting implementation : lowering complexity from $O(N^2)$ to $O(N\log(N))$
3. Image denoising with sparsity and smoothness prior



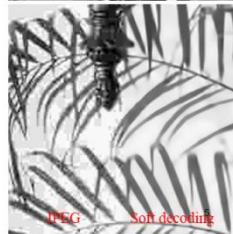
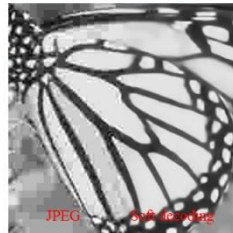
Hard Decoding vs. Soft Decoding

□ Hard Decoding

- Reconstruct DCT coefficients using the **centers** of assigned quantization bins.

□ Soft Decoding

- Find the most probable signal **WITHIN** the set of quantization bin constraints.
- **Signal priors** is used for aid
 - **Laplacian** [Lam and Goodman, TIP'00]
 - **Local/non-local similarity** [Zakhor, TCSVT'92] [Zhai et al., TCSVT'08, TMM'08] [Zhang et al., TIP'14]
 - **Total Variation** [Bredies, SIAM J. Img. Sci'12]
 - **Sparsity** [Jung et al., SPIC'12] [Liu et al., CVPR'15, TIP'16]
 - **Sparsity + TV** [Chang et al. TSP'15]
 - **Low-rank Prior** [Zhao et al., TCSVT'16][Zhang et al., TIP'16]



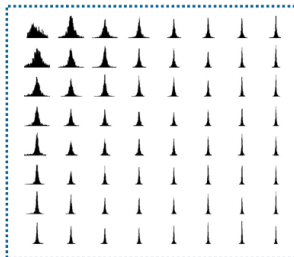


Laplacian Prior

- Q-bins: constrain the search space of individual DCT coefficients
- Laplacian Prior: states the probability density function of individual DCT coefficients

$$P_L(Y_i) = \frac{\mu_i}{2} \exp(-\mu_i |Y_i|)$$

[Lam and Goodman, TIP'00]



■ MMSE Formulation

$$Y_i^* = \arg \min_{Y_i^o} \int_{q_i Q_i}^{(q_i+1)Q_i} (Y_i^o - Y_i)^2 P_L(Y_i) dY_i.$$

■ Closed-form Solution

$$Y_i^* = \frac{(q_i Q_i + \mu_i) e^{\left\{ \frac{-q_i Q_i}{\mu_i} \right\}} - ((q_i + 1) Q_i + \mu_i) e^{\left\{ \frac{-(q_i+1) Q_i}{\mu_i} \right\}}}{e^{\left\{ \frac{-q_i Q_i}{\mu_i} \right\}} - e^{\left\{ \frac{-(q_i+1) Q_i}{\mu_i} \right\}}}$$

For higher frequencies, the Laplacian parameter is larger;
i.e., the distribution is sharper and more skewed to 0.



Sparsity Prior

□ Sparse Signal Model

$$\mathbf{x} = \Phi \alpha + \xi$$

over-complete dictionary sparse code

□ Sparse Coding

$$\alpha^* = \arg \min_{\alpha} \|\mathbf{x} - \Phi \alpha\|_2^2 + \lambda \|\alpha\|_0,$$

- orthogonal matching pursuit (OMP) [Cai and Wang, TIT'11]
- computational complexity is linear with the size of dictionary

□ Sparsity Prior

$$P_S(\mathbf{x}) \propto \exp(-\lambda \|\alpha\|_0).$$



Sparsity-based Soft Decoding

$$\begin{aligned} \min_{\{\mathbf{x}, \boldsymbol{\alpha}\}} & \|\mathbf{x} - \Phi\boldsymbol{\alpha}\|_2^2 + \lambda \|\boldsymbol{\alpha}\|_0, \\ \text{s.t. } & \mathbf{q}\mathbf{Q} \preceq \mathbf{T}\mathbf{M}\mathbf{x} \prec (\mathbf{q} + 1)\mathbf{Q} \end{aligned}$$

- *Step 1–Initial Estimation:* The Laplacian prior is used to get an initial estimation of \mathbf{x} .
- *Step 2–Sparse Decomposition:*

$$\boldsymbol{\alpha}^{(t)} = \arg \min_{\boldsymbol{\alpha}} \|\mathbf{x}^{(t)} - \Phi\boldsymbol{\alpha}\|_2^2 + \lambda \|\boldsymbol{\alpha}\|_0,$$

- *Step 3–Quantization Constraint:*

$$\begin{aligned} \mathbf{x}^{(t+1)} &= \arg \min_{\mathbf{x}} \|\mathbf{x} - \Phi\boldsymbol{\alpha}^{(t)}\|_2^2, \\ \text{s.t. } & \mathbf{q}\mathbf{Q} \preceq \mathbf{T}\mathbf{M}\mathbf{x} \prec (\mathbf{q} + 1)\mathbf{Q} \end{aligned}$$

Lemma 1: The sparsity-based soft decoding algorithm converges to a local minimum.



Limitation of Small KSVD Dictionary

- Complexity linearly increases with the size of dictionary.
- In practice, a just reasonable over-complete dictionary is used.
- KSVD Dictionary Training

$$\min_{\Phi, \{\alpha_i\}} \sum_{i=1}^N \|\mathbf{x}_i - \Phi \alpha_i\|_2^2 + \lambda \|\alpha_i\|_0,$$

Training pixel patch
DCT patch $\mathbf{X}_i = \mathbf{T}' \mathbf{x}_i$

Parsavel's theorem

$$\min_{\Phi, \{\alpha_i\}} \sum_{i=1}^N \|\mathbf{X}_i - \mathbf{T}' \Phi \alpha_i\|_2^2, \quad \text{s.t., } \|\alpha_i\|_0 \leq K$$

pre-set sparsity limit

We analyze the behavior of dictionary learning in frequency domain

Limitation of Small KSVD Dictionary



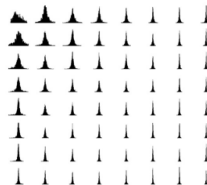
When $K = 1$, dictionary learning becomes vector quantization (VQ) design problem

- Selecting M atoms is analogous to designing M partitions

$$\mathbf{R} = \cup_{m=1}^M \mathbf{R}_m \quad \mathbf{R}_i \cap \mathbf{R}_j = \emptyset, \forall i \neq j$$

- When N tends to infinite:

$$\min_{\{\phi_m\}} \sum_{m=1}^M \int_{\mathbf{R}_m} \underbrace{\|\mathbf{X} - \mathbf{T}'\phi_m\|_2^2}_{\text{Expected square error}} P(\mathbf{X}) d\mathbf{X}$$



a product of Laplacian distributions for individual DCT frequencies

- low frequencies: decay slowly
- high frequencies: more skewed and concentrated around zero



Limitation of Small KSVD Dictionary

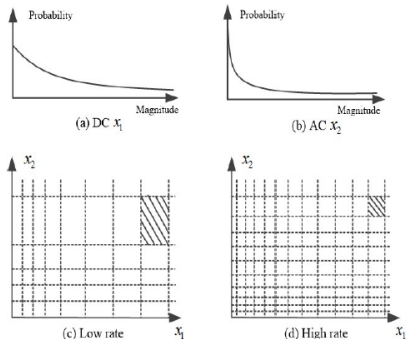


Illustration of product VQ for DC and AC frequencies

□ When the number of atoms is small

- quantization is coarser for large magnitude in AC than DC

When the dictionary Φ is small, the sparsity prior is difficult to recover large magnitude of high DCT frequencies.

□ When the dictionary is large enough

- quantization for large magnitude in high frequency is sufficiently fine.

When the dictionary Φ is large enough, the sparsity prior can recover large magnitude of high DCT frequencies well.

Graph-signal Smoothness Prior



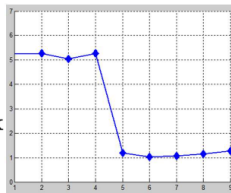
Graph Laplacian Regularizer

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = \frac{1}{2} \sum_{(i,j) \in \mathcal{E}} (x_i - x_j)^2 W_{i,j} \quad \longrightarrow \quad P_G(\mathbf{x}) \propto \exp(-\lambda_2 \mathbf{x}^T \mathbf{L} \mathbf{x})$$

Graph Frequency Interpretation

- Eigen decomposition: $\mathbf{L} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^T$
 - eigenvalues carry the notion of frequency
- Graph Fourier transform: $\mathbf{F} = \mathbf{U}^T \rightarrow \boldsymbol{\alpha} = \mathbf{F} \mathbf{x}$
- We get

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = \boldsymbol{\alpha}^T \mathbf{\Lambda} \boldsymbol{\alpha} = \sum_k \eta_k \alpha_k^2.$$



- Minimizing $\mathbf{x}^T \mathbf{L} \mathbf{x}$ will suppress high graph frequencies and preserve low graph frequencies.

- \mathbf{x} is smoothed with respect to the graph
- **PWS signals** can be well approximated by low graph frequencies for appropriately constructed graphs. [Hu et al., MMSP'14, ICIP'14]

Interpretation from the Perspective of Spectral Clustering



Rayleigh quotient
with respect to \mathcal{L}_n $\leftarrow \min_{\mathbf{v}} \frac{\mathbf{v}^T \mathcal{L}_n \mathbf{v}}{\mathbf{v}^T \mathbf{v}}, \text{ s.t. } \mathbf{v}^T \mathbf{v}_1 = 0$

- \mathbf{v} is orthogonal to \mathbf{v}_1 , according to Rayleigh quotient, the solution is the second eigenvector of \mathcal{L}_n Ln:Normalized Laplacian See p.56

The second eigenvector \mathbf{v}_2 of \mathcal{L}_n is a relaxed solution to the Ncut problem, which is PWS; if the solution becomes exact, then \mathbf{v}_2 is PWC.

- Matrix similarity transformation¹

$$\mathcal{L}_r := \mathbf{D}^{-1/2} \mathcal{L}_n \mathbf{D}^{1/2} = \mathbf{D}^{-1} \mathbf{L}$$

Random walk graph
Laplacian!

- \mathcal{L}_r has the left eigenvectors $\mathbf{V}^T \mathbf{D}^{1/2}$

$$\mathbf{V}^T \mathbf{D}^{1/2} \mathcal{L}_r = \mathbf{\Lambda} \mathbf{V}^T \mathbf{D}^{1/2} \quad \mathcal{L}_n = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^T$$

- GFT using the left eigenvectors

$$\beta = \mathbf{V}^T \mathbf{D}^{1/2} \mathbf{x}$$



Random Walk Graph Laplacian

□ We use $\mathcal{L}_r^T \mathcal{L}_r$ instead, and can derive:

$$\mathbf{x}^T \mathcal{L}_r^T \mathcal{L}_r \mathbf{x} = (\mathbf{x}^T \mathbf{D}^{1/2} \mathcal{L}_n) \mathbf{D}^{-1} (\mathcal{L}_n \mathbf{D}^{1/2} \mathbf{x})$$



$$\boldsymbol{\gamma} = \mathcal{L}_n \mathbf{D}^{1/2} \mathbf{x}$$

$$\mathbf{x}^T \mathcal{L}_r^T \mathcal{L}_r \mathbf{x} = \boldsymbol{\gamma}^T \mathbf{D}^{-1} \boldsymbol{\gamma}$$



$$\frac{\boldsymbol{\gamma}^T \boldsymbol{\gamma}}{d_{\max}} \leq \boldsymbol{\gamma}^T \mathbf{D}^{-1} \boldsymbol{\gamma} \leq \frac{\boldsymbol{\gamma}^T \boldsymbol{\gamma}}{d_{\min}} \Rightarrow (d_{\min}^{-1}) \boldsymbol{\gamma}^T \boldsymbol{\gamma}$$



Random Walk Graph Laplacian

$$\begin{aligned}\gamma^T \gamma &= \mathbf{x}^T \mathbf{D}^{1/2} \mathbf{V} \mathbf{\Lambda} \mathbf{V}^T \mathbf{V} \mathbf{\Lambda} \mathbf{V}^T \mathbf{D}^{1/2} \mathbf{x} \\ &= \beta^T \mathbf{\Lambda}^2 \beta = \sum_k \tilde{\eta}_k^2 \beta_k^2.\end{aligned}$$

- We have a graph frequency interpretation of our Left Eigenvector Random-walk Graph Laplacian (**LERaG**) $(d_{\min}^{-1}) \gamma^T \gamma$:

high frequencies of random walk graph Laplacian are suppressed to restore smooth signal \mathbf{x}

- The proposed regularizer can be efficiently computed as:

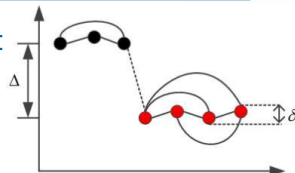
$$(d_{\min}^{-1}) \gamma^T \gamma = \mathbf{x}^T (d_{\min}^{-1}) \mathbf{L} \mathbf{D}^{-1} \mathbf{L} \mathbf{x}$$

Only adjacency matrix is involved, no need to compute other matrix

Analysis of Piecewise Smooth Signals



- 1D piecewise smooth (PWS) signal:
- A full-connected graph is built



- The normalized graph Laplacian \mathcal{L}_n is still block-diagonal
- The second eigenvector \mathbf{v}_2

1st eigenvector for e-value = 0:

$$\mathbf{v}_1 = \mathbf{D}^{1/2} \mathbf{1}$$

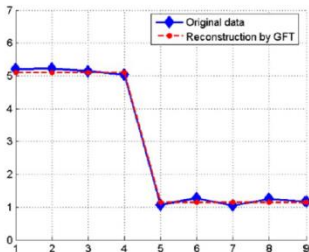
$$v_{2,i} = \begin{cases} \frac{D_{i,i}^{1/2}}{\sum_{j=1}^l D_{j,j}^{1/2}} & \text{if } 1 \leq i \leq l \\ -\frac{D_{i,i}^{1/2}}{\sum_{j=l+1}^n D_{j,j}^{1/2}} & \text{if } l < i \leq n \end{cases}$$



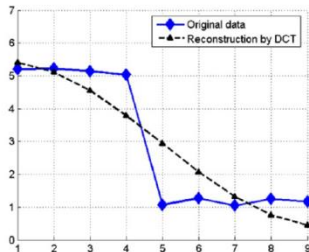
Roughly PWS

- $\mathbf{D}^{1/2} \mathbf{x}$ is also roughly PWS: $\mathbf{D}^{1/2} \mathbf{x} \approx a_1 \mathbf{v}_1 + a_2 \mathbf{v}_2$
- There is a small penalty of LERaG.

Analysis of Ideal Piecewise Smooth Signals



(a)



(b)

Soft Decoding via Priors Mixture



□ The objective function

$$\begin{aligned} \arg \min_{\{\mathbf{x}, \boldsymbol{\alpha}\}} & \|\mathbf{x} - \Phi \boldsymbol{\alpha}\|_2^2 + \lambda_1 \|\boldsymbol{\alpha}\|_0 + \lambda_2 \mathbf{x}^T (d_{\min}^{-1}) \mathbf{L} \mathbf{D}^{-1} \mathbf{L} \mathbf{x}, \\ \text{s.t. } & \mathbf{q} \mathbf{Q} \preceq \mathbf{T} \mathbf{M} \mathbf{x} \prec (\mathbf{q} + 1) \mathbf{Q} \end{aligned}$$

- λ_1 is fixed
- We adaptively increase λ_2 if q -bin indices q indicate the presence of high DCT frequencies in target \mathbf{x} .

□ Optimization

- Laplacian prior provides an initial estimation;
- Fix \mathbf{x} and estimate $\boldsymbol{\alpha}$;
- Fix $\boldsymbol{\alpha}$ and estimate \mathbf{x} .

PSNR and SSIM Evaluation



QUALITY COMPARISON WITH RESPECT TO PSNR (IN dB) AND SSIM AT QF = 40

Images	JPEG		BM3D [38]		KSVD [8]		ANCE [18]		DicTV [3]		SSRQC [20]		Ours	
	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
<i>Butterfly</i>	29.97	0.9244	31.35	0.9555	31.57	0.9519	31.38	0.9548	31.22	0.9503	32.02	0.9619	32.87	0.9627
<i>Leaves</i>	30.67	0.9438	32.55	0.9749	33.04	0.9735	32.74	0.9728	32.45	0.9710	32.13	0.9741	34.42	0.9803
<i>Hat</i>	32.78	0.9022	33.89	0.9221	33.62	0.9149	33.69	0.9169	33.20	0.8988	34.10	0.9237	34.46	0.9268
<i>Boat</i>	33.42	0.9195	34.77	0.9406	34.28	0.9301	34.64	0.9362	26.08	0.7550	33.88	0.9306	34.98	0.9402
<i>Bike</i>	28.98	0.9131	29.96	0.9356	30.19	0.9323	30.31	0.9357	29.75	0.9154	30.35	0.9411	31.14	0.9439
<i>House</i>	35.07	0.8981	36.09	0.9013	36.05	0.9055	36.12	0.9048	35.17	0.8922	36.49	0.9072	36.55	0.9071
<i>Flower</i>	31.62	0.9112	32.81	0.9357	32.63	0.9271	32.67	0.9314	31.86	0.9084	33.02	0.9362	33.37	0.9371
<i>Parrot</i>	34.03	0.9291	34.92	0.9397	34.91	0.9371	35.02	0.9397	33.92	0.9227	35.11	0.9401	35.32	0.9401
<i>Pepper512</i>	34.21	0.8711	34.94	0.8767	34.89	0.8784	34.99	0.8803	34.24	0.8639	35.05	0.8795	35.19	0.8811
<i>Fishboat512</i>	32.76	0.8763	33.61	0.8868	33.36	0.8809	33.60	0.8861	32.53	0.8496	33.68	0.8859	33.73	0.8871
<i>Lena512</i>	35.12	0.9089	36.03	0.9171	35.82	0.9146	36.04	0.9177	34.85	0.8986	36.09	0.9187	36.11	0.9191
<i>Airplane512</i>	33.36	0.9253	34.38	0.9361	34.36	0.9341	34.53	0.9358	33.75	0.9134	35.81	0.9355	36.07	0.9439
<i>Bike512</i>	29.43	0.9069	30.47	0.9299	30.66	0.9258	30.71	0.9298	30.05	0.9043	32.26	0.9372	32.55	0.9387
<i>Statue512</i>	32.78	0.9067	33.61	0.9188	33.55	0.9149	33.55	0.9193	32.53	0.8806	34.88	0.9249	34.95	0.9273
Average	32.44	0.9097	33.52	0.9264	33.50	0.9229	33.57	0.9258	32.25	0.8945	33.91	0.9283	34.41	0.9311